Policy Study No.102

Maths Problem

Can more pupils reach higher standards?

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Contents

Glossary

FIMS	First International Mathematics Study (1964)
GCSE	General Certificate of Secondary Education
HMI	Her Majesty's Inspector(s)
NCC	National Curriculum Council
SIMS	Second International Mathematics Study (1981)
SMP	School Mathematics Project
TGAT	Task Group on Assessment and Testing
TOTL	Teacher Opportunity to Learn

Introduction

IN JULY 1987 THE DEPARTMENT OF EDUCATION AND SCIENCE ISSUED the 'discussion document', *The National Curriculum 5-16*, setting out proposals for the establishment of a national curriculum, attainment targets and testing at ages 7, 11, 14, and 16. It offered the prospects of a 'proven and essential way' towards raising standards of achievement. Now, less than eighteen months later, not only has discussion been inadequate, but we are likely to see established by statute a testing system of Byzantine complexity (and commensurate expense) which, far from being based on models elsewhere, is, in fact, untried and unparalleled. Moreover, it is geared to a model of curriculum design which will not only be peculiar to England and Wales, but suffers from serious educational and pedagogical deficiencies. What went wrong, and what, within the provisions of the Education Reform Act, can be done to effect improvements?

First, it must be accepted that many questions should be asked about mathematics education as at present provided in England and Wales. Nor is there any doubt that clearer guidelines and accepted national criteria are needed. Moreover, testing has now become law, and this must be done in a manner which is both effective and economical (in terms of both financial and human resources) and which heightens pupils' levels of attainment.

It is the purpose of this paper to look closely at the problems within our educational system which call for an urgent response; and to consider the soundness of the 'remedies' recently proposed by the new National Curriculum Council. My first conclusion is that an immediate halt must be called to any further development along present lines. Study of the report of the Task Group on Assessment and Testing, of its supplementary reports, and of the detailed reports of the Mathematics and Science Working Groups[1] suggests that:

> (i) the TGAT model, as interpreted by the Secretaries of State, is based on an impossible educational assumption: that curricula can be independent of the student's age and ability. This will lead to inappropriate curricula for all levels of ability.

(ii) the TGAT model, in respect both to its principle of differentiation* by individual progress and to its requirements for testing, assessment, reporting and moderation, makes unreasonable and indeed impossible demands of the classroom teacher.

(iii) the TGAT model claims to be discipline free, i.e. it can apparently be applied to any subject. Thus all subjects are assumed to be learned in the same hierarchical way and to have a 'seven-year gap'**; this assumption is very questionable.

(iv) the use of the TGAT model forces Working Groups to give assessment procedures priority over considerations of teaching and learning: this inverts the sensible order of curriculum design.

(v) the TGAT proposals for testing are unmanageable on the resources available. They would divert too much of teachers' energies and public money from real educational needs.

(vi) there is a fundamental contradiction in using the students' assessments which are to a large degree teacher-based, in order to assess teachers and schools.

(vii) experience elsewhere would suggest that too much is hoped from a common curriculum and the imposition of external testing.

These are problems which will affect all subjects included in the National Curriculum. Specific to mathematics one notes that:

(viii) the proposals will do little towards solving the major problems of mathematical education; in particular, that of the numbers of those continuing with mathematics post-16.

(ix) there is no indication that the working group arrived

* By 'differentiation' I mean the steps taken to cope with variations in ability and attainment between pupils.

** Research carried out in England suggested that whereas an average child could perform certain mathematical tasks at 11 but not at 10, some seven year-olds can do them at 7 – and some 14 year-olds can't do them at 14 – hence the expression 'the seven-year gap'.

at any clear aims for school mathematics.

(x) research findings and comparative studies appear not to have been taken into account.

(xi) no cogent reasons can be discerned for the allocation of topics to levels, e.g. cognitive developmental constraints*, social needs or the requirements of other subjects.

(xii) unrealistic and inappropriate targets are set for teachers of high-ability, primary-school children: there is a great danger of superficial and harmful treatment of topics.

(xiii) the programmes of study provide insufficient help and guidance to teachers.

(xiv) there is no meshing with the science proposals: possibilities for mutual support have been ignored. (The TGAT model encourages subject curricula to be developed in isolation from each other.)

These are very powerful objections. The major part of the paper is devoted to substantiating them. Finally, I offer suggestions for a short-term course of action which meets the requirements of the Education Reform Act and briefly consider other, long-term action.

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* That is, research into learning warns us of psychological barriers likely to be encountered if we attempt to teach certain topics to 'average' pupils before certain ages.

2

The two major problems

There are two major 'output' problems relating to the teaching of mathematics in England and Wales: the first concerns the attainment levels achieved by school-leavers, the second the very low percentage of students who study mathematics after the age of 16. Before considering the proposals made in the last two years by the Secretary of State for Education and Science and the way in which these may be transformed into practice, the reader must understand what both these problems entail.

The problem of attainment

Is attainment lower in our schools than in those of comparable countries? Are levels of attainment lower now than they were two decades ago?

Comparing levels of attainment is not easy, for countries do not share similar syllabuses, aims, traditions or cultures. The time devoted to the teaching of mathematics may, for example, differ considerably; teaching forces may have very different professional and mathematical backgrounds; the motivation of pupils and the pressures placed on them to succeed can vary enormously from society to society. Such factors must all influence levels of attainment; yet if one sets these aside for a moment and turns to what mathematics 14 and 16 year-olds from different countries can do, then we see why we in England and Wales should be seriously concerned.

Much of the data quoted to substantiate the need to question the effectiveness of our teaching stems from the *Second International Mathematics Study* (SIMS). This comparative study carried out in 1981 was wide-ranging: for example, it investigated student and teacher attitudes, teachers' professional training, syllabus content and teaching methods, but most attention has been focused on the attainment tests it carried out at the two levels, 13+ (Population A) and 17+ (Population B). Twenty countries* participated at the 13+ level, fifteen at 17+.

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* Belgium (Fr.), Belgium (Fl.), Canada (Ont.), Canada (B.C.), England and Wales, Finland, France, Hong Kong, Hungary, Israel, Japan, Luxemburg, New Zealand, Nigeria, Scotland, Swaziland, Sweden, The Netherlands, Thailand, USA.

The tests were constructed by an international panel and because of problems of marking (especially) were all in the 'multiple choice' mode, i.e. students had to select the correct answer from five alternatives. This was a major constraint and somewhat restricted the range of mathematics, and of mathematical abilities tested. Moreover, some procrastination at the DES meant that England and Wales joined the study only when it was well under way and too late for our representatives to affect its design. So the questions asked were less well matched to the contents of our pupils' teaching than was the case in some other countries (e.g. in England, 68% were appropriate, in France 72%, and in Japan 77%). A further complication concerned the way in which the sample was constructed. In most countries schools were selected for sampling, and that was that. In England and Wales, however, schools were invited to participate and, of course, some refused. Independent and boys' grammar schools were under-represented in the sample, particularly at the 13+ level where, for instance, of the six grammar schools sampled, four were girls' single-sex, one boys' and one mixed. So the English input to SIMS had problems. Nevertheless, the results at 13+ were deeply disappointing and disturbing. Other than in a few cases, the English results tallied with those obtained by our Assessment of Performance Unit. Moreover, they confirmed the findings of other, smaller, comparative studies. Overall we were ranked 11th in attainment, being well behind Japan, Hungary and France, as the following table, giving the results of a range of developed countries, shows:

Table 1 Population A: Mean percentage of items correct

Country	Arithmetic	Algebra	Geometry	Statistics	Measurement	Overall	Ranking	Average pupil age (months)
England	48	40	45	60	49	47	11	170
France	58	55	38	57	60	53	5	170
Hungary	57	50	53	60	62	56	3	171
Japan	60	60	58	71	69	62	1	161
USA	51	43	38	57	42	46	14	170
Sweden	41	32	39	56	49	41	17	167
No. of items	46	30	39	18	24	157		

Source SIMS *(1987)*

Our results in Arithmetic and Algebra were, it will be seen, particularly weak, and the significance of the Japanese entry in the last column should not be lost.

Ten countries had participated in a similar study (FIMS) carried out in 1964 and so the opportunity was taken to compare attainment at these two different times by reusing some thirty-odd items in 1981 that had first been used in 1964. Again, the results were most disturbing. Overall, a few countries lost some ground including Japan (64.3% to 64%) and the USA (47% to 45%). But our results dropped from 52% to 44% and we were the only country to have recorded lower levels of attainment in all the four divisions: Arithmetic, Algebra, Geometry and Measurement, and Statistics. Again, some reasons can, and have, been advanced, but who can ignore such results?

What, however, has been left undone is a study of the SIMS data in detail, to see the specific lessons for us. I shall describe some possible lessons in a later chapter, without claiming that this should in any way be regarded as an adequate analysis.

The study of mathematics post-16

The data concerning the second major problem are less controversial. There is no doubt that a smaller percentage of our 17 year-olds study mathematics than in Japan, France, and the USA. The reasons for this are many, but one must surely be our idiosyncratic way of organising 16-19 education. In no other country do students concentrate so much on mathematics as do our 'double subject' students (or even, in almost all cases, our 'single subject' students); and in no other country does such a large percentage avoid mathematics altogether. One must bear these facts in mind when taking consolation for our SIMS 13+ results in those for 17+ where we came third to Hong Kong and Japan. More telling tables produced by SIMS are the following:

Table 2 Yield of high performance students

	% of sample scores exceeding 76%	Estimated number of students per 1000 of the age cohort exceeding 76%
Belgium (Flemish)	11	11
Belgium (French)	7	7
Canada (Ontario)	9	16
England & Wales	22	13
Finland	17	21
Hong Kong	56	33
Hungary	3	17
Japan	48	58
New Zealand	12	13
Sweden	16	19

Source: SIMS (1987, Ch.7)

Table 3 School participation rates at 17+

	Percent of age cohort in school	
	FIMS (1964)	SIMS (1981)
Belgium	13	65
England & Wales	12	17
Finland	14	59
Japan	57	92
Scotland	18	43
Sweden	23	24
USA	70	82

Source: SIMS (1987, Ch.8)

These tables need some interpretation. The first column of Table 2 tells us that (with the aid, like Hong Kong, of an enormous amount of time devoted in school to mathematics) a fair proportion of our 17+ A-level students scored highly in the SIMS tests. (Although the Japanese students spent less time on mathematics in school, they had on average over two hours a week coaching in the subject outside school.) *But because of the small percentage of English students studying mathematics at a high level, this proportion is only a small fraction of the age cohort* (column 2). Table 3 indicates how few of our 17 year-olds are in school (or sixth-form college). Even if one doubles these numbers to take account of those in full-time Further Education we have no cause for complacency.

Moreover, although only 12% of Japanese youth were taking the specialist mathematics course from which the sample was drawn, very many others were following other, quite high-level ones. Indeed, the proposals of the Secretaries of State relating to the National Curriculum for mathematics[2], mention that the Japanese intend to teach the calculus and other A-level mathematics to 70% of the age cohort. Meanwhile we aim to introduce 70% of our students to pi[3].

It could be argued that for 70% in Japan, 50% in Hungary and 90% in the USSR to learn calculus is overkill and that our more modest 15% is adequate. Given the present and forecast shortage of engineers, scientists and technologists, it would be hard to defend such a view. Moreover, the demographic dip means that even to maintain absolute numbers it will be essential to increase this percentage substantially.

These then are the two major challenges: to raise attainment levels, particularly in those parts of mathematics which are needed to meet the demands of responsible citizenship, and to raise the numbers of students keeping on with mathematics past 16 years old.

3
What SIMS teaches us

I have already said that it is not enough simply to say that England did badly in SIMS. The international study was a complex one which produced a multitude of data. Only to take note of league positions would be foolish. Here indeed we meet the first major lesson of SIMS. Good research practice, in education as in every other field, means (i) defining the problem to be investigated and developing appropriate tools, (ii) collecting the data in a scientific, unbiased manner, and (iii) analysing the data and presenting the results in a way which helps the taking of decisions. As we have seen, England arrived too late on the scene to affect (i), and there are doubts about the way in which (ii) was carried out. It remains only to add that (iii) was inadequately performed.

The *English and Welsh National Report*[4] on SIMS was published four years after the last of the three Japanese volumes and over two years after the US *Summary Report* appeared. Its use to the professional is restricted since it does not even reproduce all the questions: to be told that there were three items on percentages each answered correctly by about 50 per cent of the pupils tells us little unless we know what the questions were, and/or how students from other countries coped (and by 1987 such comparisons could be made). There was no attempt to analyse questions by 'cognitive'level according to the nature and degree of difficulty of the problems, e.g. were the students being asked to carry out routine calculations, to show that they comprehended and could use certain mathematical terminology, or to apply their knowledge to the solution of a problem? (A comparative analysis for the FIMS/SIMS items was carried out by the Japanese[5] who found that the English and Welsh system was, again, the only one to show a decrease in performance across the board – on computation, comprehension and application.) Much space is devoted in our national report to explaining away differences in the 1964/1981 results at 13+, but the fact that items were also repeated at 17+ is ignored. Here it suffices to say that, once more, the results for England and Wales were disappointing – although we did show increased attainment in algebra and in aspects of comprehension. Finally, our national report leaves us in the dark about how marks were obtained across the population as a whole, and where the apparent decline had occurred. For example, had the percentage of students who obtained correct marks for 76% or more of the common items declined, i.e. had there been a decline in attainment amongst our high-attaining students; or was it rather that the scores of the low attainers had decreased substantially? Such information must be available before we can set matters to right. Data supplied by Professor Postlethwaite of Hamburg University suggest that our poor performance was due as much to the relatively low attainment of our 'average attaining' children as to that of our 'low attainers'. Figures 1 and 2 in Appendix A provide sample 'profiles' of student performance in 1964 and 1981 and indicate cause for great concern about the attainment of our bottom 50% of pupils. Reliable information should, however, be available nationally; one should not have to try to piece together the English data from Japanese and German sources.

But even if reliable and detailed data concerning our performance in SIMS were now available they would, of course, be years out of date. Since 1981 the DES has paid great attention to low-attainers, for example through the publicly-funded LAMP project; and in 1988 claimed that the School Mathematics Project's Graduated Assessment Scheme has 'improved [low-attaining] pupils' success and confidence'[6]. This is only one of a number of schemes for low attainers, although probably the most popular and influential. If it *is* working, why spend money on something untried – on a scheme, moreover, which as we shall see runs counter to it in philosophy? Lesson one of the SIMS study, then, is that although 'if it is worth doing, it is worth doing badly' might justify my continuing to play the piano, it is an unsatisfactory maxim for organising educational research. The DES clearly needs reliable information on which to base its policies. To obtain this, more attention must be paid to ensuring that such research as is undertaken is adequately funded and mounted.

4

Some misconceptions removed

The SIMS attainment tests comprised over 150 items at 13+ (the numbers used and analysed varied from country to country) and some 130 at 17+. Something could be learned from each. This however, is not the place to carry out a detailed analysis. But consideration of seven items set at 13+ might help remove certain misconceptions. These seven items and the results obtained on them by English and Welsh, French and Japanese students are to be found in Appendix B on page 44. What do we learn? Arising from only this sample we note:

(a) It is not the case that if a topic is in a national curriculum it will necessarily be taught. (See various TOTLs.*)

(b) It is not the case that if a topic is in a national curriculum and is taught, then it will necessarily be learned. (See France's performance on percentages.)

(c) It is not true that Japanese pupils can carry out only routine arithmetical and algebraic manipulations on which they have had intensive coaching. The Japanese answers to item 4 provide a counter-example (and not only to item 4).

(d) It would not appear to be the case that English pupils compensate for poor attainment on formal items such as 6, by very good performance on pattern-spotting (items 4,5). (Other examples only reinforce this view.)

(e) It would not appear to be the case that, for example, scientific notation (item 6) is beyond the range of most pupils. (It appears in level 7 of the NCC proposals[7], and so would be expected of only above-average ability students at age 16.)

(f) Often the national gains in attainment made within an academic year can be very small (items 1,3,4), unless a topic has been specifically taught in that year

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* TOTLs (see p. 44) give an indication of the percentage of teachers who claimed to have taught a particular subject.

> (item 7). However – though this is not revealed by the minimal data provided here – it is not the case that once a pupil can 'do' a problem, then she/he can always do it. Thus, for example, if we look at how Japanese students tackled item 3, other data tell us that only 38.5% obtained the correct answer at the beginning of the year and at the end. This means that 80% of the students got the correct answer on at least one of the two occasions. What does this tell us about the number of Japanese pupils who 'know' how to solve such a problem, and about the 'level' they have reached?

Clearly, much more work remains to be done on the analysis of the SIMS data. Unfortunately, none of its detailed findings appears to have been taken into account when the proposals for a national curriculum were drawn up.

5
The solution proposed

The desirability of a National Curriculum was spelt out in DES's 1987 paper *The National Curriculum 5-16*. It was argued that such a curriculum would help raise standards of attainment by;

(i) ensuring students did not drop subjects too early;
(ii) setting clear objectives;
(iii) ensuring that all pupils . . . have access to 'broadly the same good and relevant curriculum'; and
(iv) checking on students' performance.

There are, of course, justified worries about students' dropping, say, science subjects too early and then finding themselves effectively cut off from certain types of employment. Yet laying down general courses of study and prescribing time-table hours, as was done in *Regulations for Secondary Schools, 1904* would suffice to secure aim (i), without the need for agreed national subject syllabuses. (The prescription of time-tabled hours was, in fact, renounced in 1907.)

That teachers should have clear objectives is, of course, essential. There is, however, no evidence to suggest that attainment will necessarily be raised at the secondary school level if these objectives are imposed nationally rather than, say, selected by heads of department from the range of those offered by existing examining boards (and which already conform to agreed national criteria).

In fact, in the case of mathematics, SIMS revealed[8] that the coverage of 13+ items tested was more uniform in England and Wales than in some other countries which already have a national curriculum.

SIMS distinguished between three curricula: the *intended*, what is to be found in national syllabuses, the *implemented*, what is taught, and the *attained*, what is learned. The three are very different! It is the case that in Japan the coverage of algebra and arithmetic is much more uniform than elsewhere (i.e. if a topic is taught by many teachers it is usually taught by almost all). But so far as geometry and statistics were concerned, uniformity in Japan was no greater than in England.[9]

The data on percentages given in Appendix B demonstrate clearly that even where the intended and implemented curricula largely coincide (as, in this instance, in France), the attained curriculum can still differ very greatly.

Clear objectives are needed *but to be effective they must be objectives accepted by teachers.* Those set by governments will, of themselves, achieve nothing. No change in practice, no change in the curriculum has any meaning unless the teacher understands it and accepts it. If a doctor gives an injection, the efficiency of the injection does not depend on his faith in the formula he has used. The teacher's faith does matter. Otherwise, at the best he will merely go on doing what he has always done; at the worst he will produce a travesty of what is intended. (The need to secure teachers' commitment to the National Curriculum is, rightly, stressed in *Mathematics for ages 5 to 16* which found the lessons observed in France owed very little to the latest national guidelines.)

Again, it is vital not only that *all* teachers have clear objectives, but that they should regularly check on their students' performance to see how far they are being realised. Yet the country whose teachers claimed to make the most use of tests, according to the SIMS study, appeared to be the USA. The TGAT Report[10] tells us of the system in Sweden where 'standardised achievement tests . . . are compulsory beyond 14, voluntary – but widely used – from about age 9 to 14'. We refer the reader to Table 1 to judge the effectiveness of these measures. Standardised tests, then, are not by themselves the answer! Testing (not necessarily of the written form) is an essential part of teaching, but let us beware the potential effect of that form of it which in the USA has led to 'dumbing down' school texts 'in which much is mentioned but little is taught'[11].

Standardised tests used as in Sweden and the USA can be seen as a form of external testing. However, it is indisputable that no developed country suffers external testing as frequently as is now proposed for England and Wales. As *Mathematics for ages 5 to 16* makes clear, in the Netherlands, 'Many pupils do not take a national examination until the age of 17 or 18', in the Federal Republic of Germany, 'there is no national examination system [up to age 16] . . . pupils' attainments are recorded in a formal system of annual reports [based on] continuous assessment and periodic [internal] testing checked and moderated by school inspectors', and in Japan 'there are no formal arrangements for testing and assessment within the national curriculum'. I have visited several developing countries where external examinations are held frequently, even annually. It is hard to believe that these have led to a raising of standards. Testing on the scale envisaged in the TGAT report may, perhaps, have something to offer. But it is clear that in no way can such a system be described as 'proven and essential'. If standards in English secondary schools have fallen in the past two decades it is in a context of more rather than less external testing. Only 312,000 (with some duplications) of the 1962 cohort were 'preparing' to take CSE or 0-level in 1966. The corresponding number for 1988 was 765,000.

Moreover, a National Curriculum demands continual national investment – together with acknowledgement of the need for its continual review. Thus, for example, the essential structure of the Italian school curriculum has not changed since the reforms of 1923; there have been periodic variations of syllabuses, but adjustments 'resulted in something not very consistent, as they juxtaposed old and new in a rather confused manner'[12]. Nearer home, we note that the inter-board core for A-level mathematics, designed in the 1970s when the flow of A-level mathematicians to university showed entirely different patterns from those of today, still awaits revision.

It is also essential that national vainglory and obeisance to trends should not lead us to totally unrealistic demands and expectations. *The Independent* of 8 November, 1988 carried a report that our HMI suggested[13] '*By 11* (my italics) pupils should be able to demonstrate knowledge of some major events of British and world history within a broad chronological structure; demonstrate some understanding of the development of British and other societies and demonstrate the breadth and richness of history by drawing attention to technological, scientific and aesthetic achievements such as social and political developments. They should also understand the history of their immediate locality'. By the same post there arrived an article on education in developing countries. It began: 'it should be noted that many underdeveloped countries have extraordinary curricula on paper, but that practical circumstances impose a more realistic implemented curriculum'. Could we bear this in mind, and might children first attain such personal achievements as learning to read and write with confidence and enjoyment before they attempt to demonstrate . . . the 'aesthetic' achievements of other societies?

6

A model for assessment

In 1987 Mr Kenneth Baker asked a Task Group on Assessment and Testing (TGAT) 'to advise on the practical considerations governing assessment within the national curriculum'. The proposals made by that committee[14] were then accepted by the Government, and the Working Groups were asked to design curricula to fit the TGAT model.

TGAT had essentially to try to solve two extremely difficult, perhaps intractable, problems. One was to devise assessment procedures which did not tend to drag down education in the manner of many standardised tests. The second was the even knottier problem of how to cope with pupils of different attainment and ability. National approaches to differentiation vary. To quote the great Russian mathematician, Kolmogorov: 'Soviet psychologists are unanimous in the opinion that all children are capable of being taught . . . capable of mastering the school material within the limits of the curriculum; and that the teacher should see to it that all pupils do so'[15]. In USSR schools there is, in principle, no differentiation made in the curriculum offered to pupils. This is still what happens officially, for example, in Hungary and Japan, i.e. the *intended* curriculum is the same for all. (According to SIMS, over 70% of schools in these countries did not set or stream, and these practices are, indeed, forbidden in Finland.) There is, of course, differentiation in practice, for less is expected of lower attaining children. (Teachers in Finland are specifically expected to cater for different ability groups within the same classroom and within the same curriculum.) In some countries, this approach is linked with the use of pupil retention, i.e. lower attainers are required to repeat a year. This is not universal practice by any means; there is, for example, almost no use made in Japan of acceleration and retention.

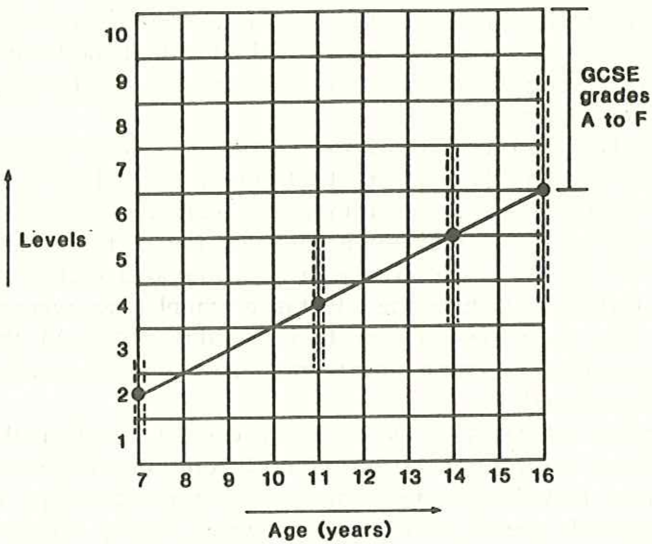
An alternative approach to differentiation is to have different curricula for different schools or for different groups of children within the same school. This is done in, say, the Federal Republic of Germany[16]. It is a system with which we in England are familiar.

Both of these systems have shortcomings. Children do differ in ability and it sets extra problems for teachers to deal with mixed-ability classes. The needs of both the brighter and the less able pupils are often ignored. On the other hand, strongly differentiated curricula make transfers between schools and groups difficult, but by no means impossible, to accomplish. There is the risk of premature labelling leading to self-fulfilling judgements, and to the formation of demoralised 'sink' groups. TGAT and the Secretaries of State have opted for neither of these courses. Indeed, Mr Baker and Mr Walker have asked that 'the attainment targets and programmes of study . . . should apply without modification to all pupils'. This does not mean, however, that we shall follow the USSR model. For TGAT has based its approach on 'the widely recognised seven year gap in attainment . . . at age 11'[17] (see p.7). Just how 'widely' this seven-year gap is recognised is not quite clear, nor why it should apparently apply to all subjects[18]. Certainly, it does not seem to be recognised in curricular terms by the Japanese, the French, the Hungarians. Yet we in England have chosen to legislate for it – and for its widening to ten or more years at age 16+. Little attention appears to have been paid to finding ways to reduce it. What, then, TGAT has proposed is that assessment and reporting should be via 'profile components*' (each individual subject would have a small number – preferably no more than four); that each profile component should contain a number of attainment targets (varying in the proposals to date from 3 to 17+) and that ten 'levels' should be defined within each attainment target. Children are expected to progress through these levels at their own rate according to what appears to be becoming a key guiding graph[19]. This graph incorporates a universal description of the learning process, and has the apparent advantage of applying, amongst others, to Mathematics, Science, English, Home Economics, Religious Education, French, and, presumably, French cricket. It is making a disciplinary-free statement and assumes that all disciplines will be learned in the same hierarchical way; a very questionable assumption indeed.

* A century ago these were called, more simply, 'algebra', 'geometry', 'arithmetic' and 'higher mathematics' – in the ambit of this paper's discussion. In modern educational jargon, they are groupings of attainment targets within each subject, for the purpose of reporting on the pupils' progress.

Figure 1

Sequence of pupil achievement of levels between ages 7 and 16



The bold line gives the expected results for pupils at the ages specified. The dotted lines represent a rough speculation on the limits within which about 80% of the pupils may be found to lie.

The curriculum, then,is not to be geared to age or ability – our children are to be treated like trams, and will follow the same tracks according to their rate of learning. Thus, presumably, 'know about a variety of methods of birth control' which is a level 6 attainment[20] could be encountered in primary school by a sufficiently (intellectually) advanced student, but would not be met (or not be expected to be understood) by some 35% of low attaining 16+ school-leavers*.

* Birth control has now (December, 1988) been removed from the National Curriculum because of religious objections. Nevertheless, it may still serve to illustrate the fundamental weakness of TGAT.

This example should make us stop and think seriously about the validity of the model, but let us phrase the objection in a more general way; does it make educational sense that a low-attaining fifteen year-old and a high-attaining nine year-old should follow the same curriculum? Is the fact that the former student is nearing the end of his or her compulsory education to pass unrecognised?

The TGAT model expects and encourages a wide spread of ability at age 16. By so doing it effectively limits those who will be properly prepared to continue with mathematics beyond 16+, for what encouragement is given to the middle ability student? In Japan, as we have seen, it is intended that some 70% of students will study the calculus in school. If we were to aim for that same percentage, then according to TGAT the laggards would be reaching the integral calculus when they were about 23 years old.

Agreed, of course, that the social context in which the Japanese are educated is vastly different from ours. Pressures there mean that the average school pupil from about age 11 upwards will spend two hours a week being coached mathematics in a crammer. We cannot expect to replicate that even if we wanted. Nevertheless, the ability of ordinary schoolchildren to learn mathematics has been demonstrated. It may be that we believe that the Japanese are by nature intellectually superior to us. But if we do not accept this then we must put down differences in levels of attainment to other, non-cognitive-psychological factors. The curriculum must be planned in recognition of this.

Still another objection to the TGAT model is that it is concerned primarily with assessment. Accordingly, the levels to be found in the mathematics proposals are often levels of understanding. To translate these into teaching and learning levels could be pedagogically disastrous. This problem is recognised[21] but is not taken into account enough in the programmes of study. For example, the attempt to fit probability into the TGAT corset[22] is open to ridicule (we shall return to this later). This is not to deny that there are possibilities, indeed a need, for propaedeutical work on probability to be included in primary school curricula. The Hungarian national curriculum[23] demonstrates this well. Yet to tie work to such attainment targets as are proposed by the mathematics working group would be potentially calamitous.

The second problem which TGAT set out to 'solve' was that of assessment procedures. To describe their proposals as 'complex' would not do them justice. Some idea of the potential cost is obtained in paragraph 211 of the TGAT Report:

> Apart from modifications which might arise for GCSE, the new work required for age 14 would imply a training operation roughly comparable in scale to the GCSE programme, perhaps greater because . . .

Recurrent as well as initial costs could be greater. Nor are such costs only financial. A recent report on the implementation of GCSE[24] tells of the cost in staff time: 'the workload on senior staff has increased'[25], 'increased demands on [schools examination officers]'[26] 'increased commitments [for heads of departments] heavily involved in the arrangements for marking, recording and moderating'[27], 'other teachers also have had to cope with an increased workload . . . almost all have been hard pressed . . .'[28].

We are heading for a very great shortage of qualified 11-16 mathematics teachers. Is it sensible to double the assessment burden already placed on them? One finding of SIMS was that when teachers in England meet they spend a greater proportion of time discussing administrative problems than do teachers elsewhere (four times as much as do the Japanese) and that the ratio of time spent on discussing administrative affairs to that spent on discussing teaching content and methods is of the order of 3 to 1.

The TGAT proposals at 14+ would worsen this bias. The Head of Department, who in theory should be the most competent, experienced and useful subject teacher in the school, will become almost a full-time administrator and examiner. Does this really make sense; is it truly the way to raise standards? In a system which, according to the Inspectorate, lacks books[29] and equipment[30] how can it be expedient to spend vast sums of money on yet more assessment?

7

Mathematics for ages 5 to 16

The case against conventional timed examinations has been argued often: candidates merely regurgitate old notes, the pressures of time militate against sensible, original thought, what a candidate writes is frequently self-contradictory since he or she will not have the time to read it over . . . In effect the mathematics working group itself had imposed on it a timed examination and had to satisfy constraints for which many of them had no sympathy. I believe I know the majority of them too well to imagine that this document represents a true consensus of what they think mathematics education is about and how we should best act to improve it.

There are many reasons why the document is a depressing one. First, it does not appear that the Group had any agreed conception of the proper aims of school mathematics. The overall *impression* on reading the report is that for practical purposes the committee merely fleshed out the TGAT model and produced the 'map' of the mathematics curriculum given on pp 71-76. This, of course, is essentially what will go into Parliamentary Orders, not the fine words and caveats. It is also this map which will form the basis for the testing.

Because of the model imposed upon them, the working group, therefore, felt no obligation to ask 'what are the special needs of the low attainer and, indeed, of the bottom 50% of the age range?' How can we ensure that these either leave school ready to face the mathematical challenges of employment and citizenship, or are encouraged to pursue the study of mathematics within vocational education? The group gave only pious statements such as 'All pupils should, *at their own level* (my italics), be able to use mathematical language fluently, recognise patterns, make generalisations, model situations, make conjectures, attempt proofs and perceive their relevance, criticise arguments, formulate, tackle and solve problems and acquire the ability to recognise a mathematical concept in, or to extract it from, a given situation'[31]. This is hardly a stout response to the messages of SIMS, the Assessment of Performance Unit, Concepts in Secondary Mathematics and Science, . . .

Neither are we explicitly told on what basis the levels are assigned. Is it to match cognitive development, social needs, the requirements of other subjects?

Let us take one example, scientific notation, e.g. expressing 22731 as $2{,}2731 \times 10^4$. This is level 7[32], i.e. it will probably be covered by only about half our school leavers. Yet if we look at SIMS 6 in Appendix B we see that 64% of Japanese pupils could answer that particular question at 13+, and that the topic appeared to be taught to over half of English 14 year-olds in 1981. Why then is this level 7 work? Is it because we believe it to be too difficult for most of our pupils to grasp, or is it that the group attached less importance to learning scientific notation than to exploring the traversability of networks (i.e. finding a path round a network which involves traversing each section once and no section twice) (level 6)? Do they not wish students to use calculators intelligently? The scientists, incidentally, expect the use of 'scientific notation' in levels 4 to 7[33]. Exactly what the scientists mean by this is open to interpretation, since they give no examples. (At least this spares them the embarrassment of supplying the same example (drawing a graph to convert Fahrenheit to Celsius) at *two* different levels[34], but it hardly helps the interpreter of the science levels.)

Yet another failing of these two reports is that they do not mesh. Thus, to take one of a host of possible examples; 'Read a temperature scale' is level 3 mathematics, whereas 'to know how temperature is measured' is level 4 science.

The two groups did not come together to discuss their work: indeed, they were actively discouraged from doing so. The result? There are inevitable mismatches; but, more importantly, science appears to be presented largely in a qualitative manner. There is little indication that many of science's major triumphs arise from the use of mathematical models. The possibility of building upon and so reinforcing students' mathematical knowledge appears to have been discarded. This is *not* the way to improve mathematical attainment or to provide motivation for the study of mathematics.

Has the reader patience to continue this dismal catalogue? The geometry, pp. 35-36, is more in the style of SMP Books 1-5 (published in the 1960s) than of those texts which have replaced them. (There is no international consensus on the content of school geometry, and what it is felt *essential* for every student to study in geometry is rather limited. Does it make sense then to stipulate a national curriculum post-level 5? Will not the backwash of GCSE, with its varied options, suffice?) This, and the practical examples provided, hardly represent the best in current practice.

Not enough thought appears to have been given to the presentation of the curriculum: for example, do we in England introduce formal material on topics too early; do we spend enough time on establishing foundations? As indicated earlier, evidence is growing that pupils are slow to learn, if they do not understand the first time round. The 'spiral' curriculum which does not build on knowledge gained, quickly degenerates into going round in circles.

There is also a tendency to translate two or three worthwhile lessons into compulsory topics in the national curriculum which, when tested, will lose most of their pedagogic benefits. One such is traversability of networks. For any student to explore this *could* be rewarding, but testing would destroy such benefits and result in the rote learning of rules of no great worth.

Here I am not opposing rote learning of tables, formulae, definitions and results which have general applicability. But rote learning of specific facts or techniques solely for assessment purposes, and even worse, rote thinking, should have no place in a national curriculum. An example that could soon degenerate into the latter is provided by 'use the difference method systematically in exploring the pattern of a sequence'[35]. The example given is 2, 5, 10, 17, 26. A purely rote method of continuing this sequence is given. Yet expanding $(x + 1)^2$ does not appear until level 8, i.e. the student has no mathematical means provided for seeing why this particular piece of rote learning should be valid and can be generalised. In effect, what can be seen as rote subtraction and addition (level 3) or exploration and deduction from known mathematical knowledge (levels 8-9) has been averaged out to keep children occupied during a mathematics lesson. The question must be asked again: is it really more important for a low attainer to do this kind of work than to learn about scientific notation? Some of these details might be rectified in the revision process now under way – but still within an unsatisfactory framework.*

Now to take another cause of grave disquiet – one apparent benefit of the TGAT proposals is that the bright child will be encouraged to move ahead. It is envisaged[36] that perhaps 10% or so of children in the upper year of primary school will reach level 6 work. In mathematical terms this means, for example, trial and improvement methods for the solution of linear and simple polynomial equations, simple inequalities, randomness (level 5), bias (level 3), probability scales (level 4), distinguishing between *a priori* and *a posteriori* probabilities (level 5), and independent events. Who is going to teach this and who is going to learn it? When SMP began in the 1960s it introduced probability in Book 3 of its grammar school course. Does any other country have such inflated ideas? I might believe there was hope if the working group had come up with a valid example of the technical notion of bias[37] or if GCSE Groups did not set such questions as:

> John cycles to school on average 3 days out of 5. . .
> Bill cycles to school on average 2 days out of 5. . .
> Find the probability that on a certain day they will both cycle to school.

(Here the student is asked to apply rote methods (thinking?) rather than commonsense to the problem – are the two events independent and unaffected by the weather?)

The introduction to the primary curriculum of yet more topics taught in a superficial, possibly incorrect manner is hardly likely to help improve matters. There is plenty of good work which bright students could do on the material contained in earlier levels which would give them a better foundation for secondary school, and which would not add to the problems of teachers there, by their feeling obliged to repeat material which pupils believed they had already covered before. The linking of student assessment to that of schools would serve only further to encourage schools to undertake inappropriate level 6 work and assessment.

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* Brief comments on the revised proposals are included in the Postscript (p. 51)

Little attempt appears to have been made to learn from other countries or even to learn from the experiences of those who had developed curricula over many years in this country. As too often in educational circles, action took precedence over acquisition of knowledge. Thus, members of the committee visited Japan only after the main body of the report was completed. Their response to Japanese standards – 'We believe that setting attainment targets at 10 different levels will help stretch pupils of all abilities'[38] – speaks for itself. The number of levels is of no consequence; what affects attainment is what the targets are and the percentage of students expected to achieve them. More detailed criticisms of the Working Group's proposals could be given, but this is not the place for them. It is enough if I have caused serious reservations about the validity of much that is contained in the published proposals. Unfortunately, most of the responses that I have seen, because of the official pro-forma supplied and the specific questions posed by the Secretaries of State, would appear to have concentrated almost entirely on whether or not Profile Component 3 (that relating to 'Using Mathematics', Communication Skills' and 'Personal qualities') should be retained. This, unfortunately, has deflected serious consideration of the proposals and their likely effects. Certainly, teachers should try to cultivate the growth of those qualities described in Attainment Targets 13 and 14; and it is reasonable to ask teachers to report on them. Whether one can test them independently of Profile Components 1 and 2 is not so clear, nor whether one can successfully teach and learn these qualities in isolation*.

* Perhaps the greatest weakness of Profile Component 3, though, was that it was impossible adequately to describe its objectives using only the simplistic syntax of the TGAT levels. (The revised Attainment Targets 1 and 9 serve only to reinforce this view.)

8

Implementing the National Curriculum

It is worth noting that the mathematics proposals mentioned textbooks only when describing visits by the group to other countries, referring in particular to the care taken in Japan to ensure that new ones are prepared prior to any attempt to implement change in curricula. By contrast our attitude appears to be that once we have well-defined targets and tests (and teachers have been supplied with one example per target), then everything else will follow.

The national curriculum is apparently to be implemented at 5+ and 11+ from September 1989. What can this possibly mean, in sensible classroom terms? What assumptions, for example, are secondary school teachers to make about the material available to their 11+ pupils? It could be argued that existing textbooks and classroom materials will suffice. If so, what has the fuss been about? What has been gained? Yet, if they will not, how is the teacher to cope? (We cannot, of course, assume that all the teachers of 11+ mathematics classes will be qualified mathematicians nor that they devote all, or even most, of their time to teaching the subject). The proposals for in-service training lack conviction: who are the 'experts' who will give them? The amount of such training to which the ordinary teacher will be exposed is inadequate, particularly if he or she will have no new classroom materials to fall back upon.

Yet textbook authors and publishers are faced with an impossible task if the attainment target levels are to be the subject of continuous revision. The only feasible solution would be for them to design modules making as little reference as possible to other topics at that or the preceding level. That would certainly be some safeguard against sudden redundancy – but the economy would be achieved at the cost of revision, reinforcement and coherence. Learning would be the loser. The Japanese example of regular, ten-year, well-planned curricular revision is worth emulation – or at least consideration. Let us anyway urge as few changes as possible in the next three or four years during which small-scale experimental development work could be mounted, prior to a major overhaul offering some degree of longevity.

At least, before any further steps are taken, could somebody of some experience plan and cost the in-service education which will be needed by teachers in junior and middle schools? These are the ones who will be hardest hit and on whom the major burdens will fall; for, in particular, they will be asked to teach many mathematical topics which no other country requires at this level of education.

Further detailed objections are superfluous. What must be stated outright is that the proposed timetables ignore the lessons of thirty years of curriculum development and in-service work. The timetable, like that for the introduction of GCSE, invites first chaos – the hurried creation by individual teachers of a *modus vivendi* – and then lacklustre acceptance of second-best routines.

9

A short-term solution?

The Education Reform Act provides for the establishment of a National Curriculum with appropriate attainment targets, programmes of study and assessment arrangements at the four key stages ending when pupils in a class are 7, 11, 14 and 16. Any proposals for change must work within that framework.

A National Curriculum has been deemed to be the first priority, i.e. the definition of the core and other foundation subjects and a determination of how timetabled hours are to be allotted (if, in fact, these are to be determined centrally). At the moment[39], the amount of time available for mathematics teaching seems yet to be agreed. It is unclear how one plans a curriculum when the time available might be increased by 25% or decreased by 20%; and it is paradoxical to ask 11-16 mathematics teachers to raise standards and take on board a vastly increased assessment workload, while simultaneously cutting their teaching time by a fifth.

Next, decisions have to be taken on how differentiation is to be achieved, i.e. how are schools to cope with pupils of different abilities and attainments? I have argued that the TGAT model is educationally indefensible. In effect, many schools nowadays opt for a model based on individually organised work or (better) group work until the age of 13 or so; and follow that by differentiation into three more clearly defined groups, each with its own appropriate curriculum. This is a model used in other countries (although sometimes different groups may be allocated to different schools). It might not be ideal but it is preferable to that encouraged by the TGAT proposals.

For mathematics this would require a National Curriculum which defined the 'basics' common to all groups, and those which were relevant to each group separately. I do not mean 'back to basics' but 'forward to basics'. Certainly, the NCC proposals do not clearly indicate what the basics common to all groups might be, other than to say 'at the last stage of compulsory education, every pupil should have developed a thorough understanding of whole numbers, decimals and percentages and an ability to use them with confidence'[40]. Their inclusion is to be welcomed, but it is strange that the only unequivocal requirements in the 14-16 programme should be 'old' basics. Are these the only 'priorities'? If so, why bother with a national curriculum, or even allot 10% of school time to mathematics? If not, can we have clearer statements of what we wish different types of students to understand? Put another way, what message is the ordinary teacher expected to take away from the programmes of study? For example, 'The use of calculators . . . is to be encouraged, but there are clear dangers in undue concentration on calculator work[41]' is no help at all.

In the short-term, however, the established National Criteria for GCSE could suffice at 16+, provided that they were supplemented in some way for those not aspiring to GCSE but currently following the earlier levels of one of the graduated assessment schemes. Immediate action would need to be taken at 11+. Again national criteria would be needed. Here the NCC proposals provide some help. However, it would be more profitable to follow practice elsewhere (and indeed current practice in England) by indicating first to what mathematics and activities children might be exposed and with what objectives, before guidance was provided on the levels of attainment to be expected. The problems of differentiation will have to be faced, but proposals for their resolution should be based on educational knowledge and objectives rather than on conformity to an imposed system of assessment.

What of testing? The TGAT proposals are basically unmanageable. What might sensible priorities be?

GCSE is now with us. The first priority in the 11-16 range must be to make it work more successfully. In mathematics it often operates through a four-in-line system. That is, weaker candidates take papers 1 and 2 which are easy but do not allow the entrants to obtain high grades; candidates of average ability take papers 2 and 3 and, provided they obtain high enough marks can obtain a C; the higher attainers take papers 3 and 4. According to the recent HMI report:[42]

> At some mathematics award meetings a higher mark was required in the higher level paper than in the intermediate level in order to achieve a grade C[43].

That sentence should be digested slowly. For we were told in the same paragraph that,

> candidates . . . entered for the higher level papers, often inappropriately. Candidates who were unable to cope with the highest level papers in mathematics would have stood a better chance of success at intermediate level.

That, in view of the earlier quotation, cannot be denied. It does, however, suggest that the examiners might also be to blame. Some action, certainly, is required. Moreover, what will be the effects on recruitment to A-level and on gender bias, if schools take the advice offered by HMI?

And what of moderation, i.e. methods for eliminating differences in marking standards between schools? The letters that I see are, naturally, those of complaint. The problems will not decrease as the schools submitting coursework increase. (Appendix H of the TGAT report suggests how moderation might be effected. It describes its proposals as speculative: 'over-optimistic' might be a more accurate term.) Close attention should be paid to this aspect of GCSE examining[44].

Another problem relates to comparability between examining groups. Much work is to be done if teachers' faith in the examination system is to be regained. For example, one school entered candidates for two groups. With Group A one candidate (out of 33) received a grade lower, and 22 received grades higher, than the school predicted; while with Group B the position was reversed; of 146 candidates, 10 were upgraded and 63 downgraded (including 16 who entered for higher grade papers and were unclassified). What does this tell us about comparability of standards between Groups? Having seen past records from the school, I am forced to believe that the HMIs were unjust in putting all the blame on teachers for the failure rates on the higher grade papers.

Then coursework. The HMI quote with approbation work done by a low-attainer on practical arithmetic. Alas, I can also cite examples of high-attainers being assigned work similar to that done thirty years ago by 13+ year-olds in good secondary modern schools. This is not to ask for coursework to be abandoned. But it must be monitored much more closely, and be the object of properly mounted research studies. Schools must not be allowed to slip into unthinking routines, but be encouraged and helped to make it a valuable component of education.

All this will take time and resources, both financial and human. These should not be diverted to setting up a supplementary examination at 14+. The influence of a good 16+ examination, felt throughout the system, should largely suffice at this level. What is needed is some check on the lower attainers. Are they going to leave school equipped with minimal, basic needs? Perhaps, then, all pupils should be asked to take a – genuinely basic – numeracy test (with mental, oral and practical components) at that stage, the aim of which would be to see whether they should proceed to the lower levels of one of the Graduated Assessment Schemes intended to provide that basic kit needed for citizenship and employment. Such testing should be entrusted to individual schools and teachers (although items would have to be taken from a central bank, set procedures followed, and written scripts kept for moderation by HMI, LEA advisers or others).

Some quarters might well object to this; yet there is no escaping the fact that a centrally controlled monitoring examination poses tremendous administrative problems. Moreover, although one can legislate for an examination to be set, one cannot ensure that 14 year-olds should take it. Temptations to be absent on the day would be great, since passing would bring no clear external recognition, and, conversely, failing no internal sanctions. Further, if schools were to be publicly judged on their success rates, it would scarcely encourage them to see that low attainers were present. So it not only makes educational, but also administrative, sense if teachers assume responsibility for 14+ testing.

Detailed results would have to be returned to the LEA or other authority, and to the providers of the item bank, so that national attainment could be monitored, the item bank appropriately adjusted, and management decisions made at a local level.

We see then that assessment at 14+ and at 16+ should not follow the same pattern. Different kinds of information are required at these two age levels and different types of people are interested in, and will act upon, the results. Similar distinctions apply at 7+ and 11+ and it is, therefore, toward this latter stage that most attention should be directed. A minimal test at 14+ should suffice because of the backwash of GCSE and because the student will almost certainly be in the same school at both 14+ and 16+. There will be no such backwash effect at 11+; and for most children the end of this stage will be at or near the time of transfer to a new school.But testing at this level *must* be differentiated. There are several ways in which this could be done, some of which are explored in the NCC proposals. There is also much LEA experience and several research studies on which to build. Once again, there is a need to involve the teacher in the assessment process. Help must be given through the provision of standard tasks and item banks. But the emphasis above all must be on teachers accepting objectives and themselves assessing to what extent these are being attained. Here we see the need to keep student assessment independent of teacher assessment. We must not have a re-run of 'payment-by-results': the teacher must cast himself as the enemy of ignorance and of any student's lack of progress, rather than of the external examiner. If we hope to effect improvements through external assessment and tests alone, we shall fail. As Matthew Arnold wrote over a century ago, such measures

> give a mechanical turn to the school teaching . . . and must be trying to the intellectual life of the school . . . In the game of mechanical contrivances the teacher will in the end beat us . . . by [getting] children through . . . the examination . . . without their really knowing anything of these matters.

Yet, if primary-school teachers are to act as reporters and assessors, then other demands made upon them must be trimmed. As new Working Group reports appear with yet more attainment targets to be assessed, it is easy to see how the present proposals will overload teachers. Assessment must not degenerate into such routines as the ticking of pro-formas to answer questions like 'does the pupil know there is a variety of weather conditions?'[45]

There *is* a need for reporting at 7+ (i.e. to parents and those within the school system). Here, the suggestions on 7+ assessment in the NCC document have much to offer. Attention, however, must be concentrated on ensuring that pupils possess the basic mathematics and English on which they can build; and that those who commence school able to read fluently and with a modest grasp of arithmetic do not spend two years on colouring and other time-filling occupations. Testing science attainment at the early levels might well serve only to add extra confusion. It would be much more profitable if recent joint work by the professional associations for English, Mathematics and Science were built upon. Guidelines and teaching suggestions prepared by them should ensure that educational needs are met without inhibiting and time-consuming assessment.

There is a danger that, as a result of current proposals, what is most admired in English education might suffer, whilst that most in need of strengthening will be little improved. It must not be forgotten that many features of our educational system at all levels *are* admired in other countries, and rightly so. Much of our primary school work is a just source of pride. Our better secondary school mathematics materials are adapted, translated and read overseas because of the concern they show for capturing the interest of students, and for relating school mathematics to the world around them. Our better software in the field of mathematics education is unrivalled. We must not kill opportunities to excel, when we try to banish unacceptably low levels of expectation in other parts of the system.

10
Looking further ahead

Some years ago, I wrote that 'one cannot talk truly about a national curriculum , for it depends upon individual teachers, their methods and understanding, and their interpretation of aims, guidelines, texts, etc. The part played by the individual teacher must, therefore, be recognised'.[46]

Good teaching depends principally upon good teachers – not good syllabuses, materials or assessment procedures, although all these can assist, and may sometimes compensate, in small part, for a weak teacher force.

There is now a great shortage of such teachers and it is essential to ask what effect the current proposals will have in attracting or deterring people from entering the profession. Will secondary school teachers with imagination be likely to make a bee-line for either the independent sector (which will be spared this 'proven and essential way towards raising standards') or sixth-form colleges? Somewhat belatedly, the Education, Arts and Science Select Committee of the House of Commons is soon to investigate 'the supply of teachers for the 1990s'. In one sense, as a recent study commissioned by the Engineering Council[47] has made clear, the position (if by that we mean supplying enough trained and competent mathematical teachers) is irremediable. Palliative tactics, should, therefore, be considered immediately – particularly because for a period of some five years some slack will remain in the system. So the opportunity exists for designing a co-ordinated system of in-service education away from the day-to-day turmoil of the classroom. To fritter away this period trying to introduce a system which holds little prospect of long-term beneficial effects, would be disastrous.

Teachers must be strengthened. The uphill battle to regain lost status must be begun, working conditions must be improved, and levels of professional competence must be raised. This means making teaching more attractive, less repellent. We must retain more existing teachers and win back some who have left the profession. It is noticeable, if not surprising, that those countries in which teachers are undervalued performed badly on SIMS. Not only this, but these same teachers reported that the time they spent attempting to maintain discipline in classroom was considerable.

Social attitudes cannot be changed overnight, but we must properly identify major problems within our educational system and attempt to tackle these head on, rather than put our trust in untried, expensive and contentious remedies.

One major problem touched on above relates to the education of 16-19 year-olds. It is astounding that we should even contemplate studying the 5-16 curriculum in isolation from the education provided in sixth-forms, sixth-form colleges and colleges of further education. But the result of our so doing is demonstrated clearly in the mathematics proposals: there are no clear goals and when the whistle blows at the end of the last school term, sixteen year-olds put down their pens, pocket their calculators and wander off. But to where? To do what?

This problem must be tackled forthwith; time scales in education are very long. Thus, for example, the CTCs will provide perhaps 20 extra technology and engineering graduands in 1998, rising to at most 100 in the year 2000. The DES idea that schools will drift into a more widely-based sixth-form curriculum in their own time, must be looked at in the light that it took thirty years for them to change from separate to combined arithmetic and algebra papers at O-level.

If we are to achieve parity of attainment with more successful developed countries, and England is to equal them in the number of pupils it puts through the educational system, then thought, planning, consultation, and action are needed urgently.

Summary of proposals

In the short term we ask to:

i) strengthen GCSE mathematics; detailed monitoring and properly mounted research studies are urgently needed;
ii) establish a minimal 'basics' test at 14+, intended to provide formative * evaluation for the bottom 40% or so of the attainment range;
iii) develop national criteria and assessment procedures for the key stage ending at 11+; and,
iv) develop assessment and reporting procedures in English and Mathematics at 7+, and encourage the preparation of cross-curricular guide-lines, activities and materials at this 'key' stage.

In the longer term we wish to:

i) prepare revised national criteria at 11+ and 16+ with detailed suggestions for programmes of study and/or guidance for teachers; bearing in mind the post-16 education and needs of different groups and types of student – and the necessity to develop more cross-curricular links; and
ii) mount research studies into the effectiveness of different teaching patterns, e.g. is it the case that not enough time is devoted to the learning of topics the first time that they are formally met by students?

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* Evaluation can be divided, roughly, into two types: 'summative' which classifies and certifies, and 'formative' which seeks to identify necessary remedial measures.

Appendix A

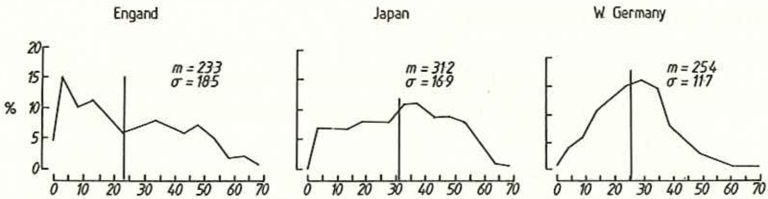


Fig. 1

Figure 1 relates to data at 13+ obtained in FIMS[48] and shows the percentage of the sample obtaining a particular mark. The average mark is denoted by m, and the standard deviation by σ. Note that England and Wales exhibited a greater spread of ability than did Japan and West Germany. We had a greater percentage of 'high attainers' than these countries, but this was more than counter-balanced by the weak performance of our 'low attainers'.

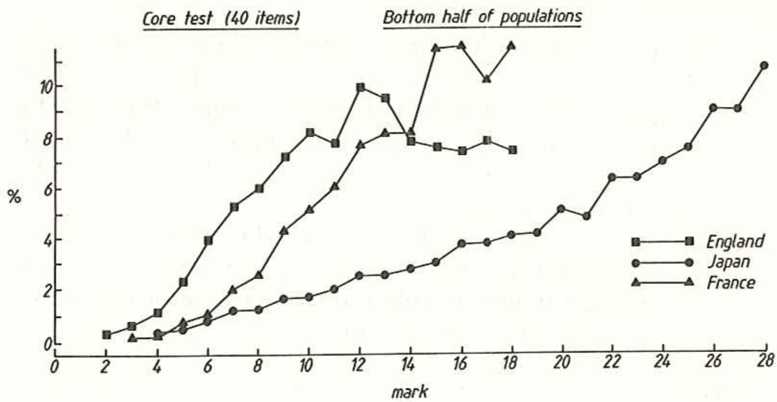


Fig. 2

Figure 2 gives data from SIMS (1981). This relates only to the bottom 50% of the population. Note the big difference between attainment levels in Japan and in England – and that between France and ourselves.

Appendix B

Seven typical SIMS items

The two entries for Japan arise from the fact that it carried out a longitudinal study, i.e. the same tests were administered at the beginning and at the end of the school year. (Thus, the Japan (1) pupils had an average age of about 154 months as against the average age of English and Welsh (ENW) pupils of 170 months.) The TOTL column gives the percentage of teachers who said that students had been taught the topic before they took the test. The country ranking relates to the twenty countries listed on p.8.

Item 1

30 is 75% of what number? (A) 40*, (B) 90 (C) 105 (D) 225 (E) 225

Country	Percent correct	TOTL	Percent omitting	Country ranking
ENW	48	81	6	9
Japan (2)	47	99	4	12
Japan (1)	46	–	5	–
France	36	91	20	18

Item 2

20% of 125 is equal to (A) 6.25 (B) 12.50 (C) 15 (D) 25* (E) 50

Country	Percent Correct	TOTL	Percent omitting	Country ranking
Japan (2)	62	100	2	5
Japan (1)	52	–	3	–
ENW	45	85	3	17
France	41	97	19	19

Item 3

In a school election with three candidates, Joe received 120 votes, Mary received 50 votes, and George received 30 votes. What percent of the total number of votes did Joe receive?

(A) 6/10% (B) 40% (C) 60%* (D) 80% (E) 120%

Country	Percent correct	TOTL	Percent omitting	Country ranking
Japan (2)	60	100	2	3
Japan (1)	59	–	3	–
ENW	48	77	2	10
France	42	94	15	15

Item 4

Matchsticks are arranged as follows

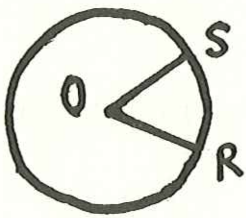
If the pattern is continued. how many matchsticks are used in making the 10th figure?

(a) 30 (B) 33*, (C) 36 (D) 39 (E) 42

Country	Percent correct	TOTL	Percent omitting	Country ranking
Japan (2)	62	44	1	2
France	60	23	6	3
Japan (1)	58	–	2	–
ENW	52	44	2	7

Item 5

The length of the circumference of the circle with centre at O is 24 and the length of arc RS is 4. What is the measure in degrees of the central angle ROS?



(A) 24 (B) 30 (C) 45 (D) 60* (E) 90

Country	Percent correct	TOTL	Percent omitting	Country ranking
Japan (2)	74	93	3	1
Japan (1)	66	–	11	–
ENW	34	27	9	11
France	29	61	28	16

Item 6

3.23×10^4 is equal to

(A) 0.0000323, (B) 3.23000, (C) 32,300 (D) 323,000* (E) 32,300,000.

Country	Percent correct	TOTL	Percent omitting	Country ranking
Japan (2)	64	78	2	1
France	62	97	4	2
ENW	38	56	3	15

Item 7
If P = LW and if P = 12 and L = 3 then W is equal to
(A) .75 (B) 3 (C) 4* (D) 12 (E) 36

Country	Percent correct	TOTL	Percent omitting	Country ranking
France	89	97	3	1
Japan (2)	81	94	2	2
England 1964	64	–	8	–
England 1981	56	77	5	15
Japan (1)	51	–	21	–

Omission rates have been included in these tables since they differ substantially from country to country (English pupils rarely allow ignorance or lack of preparation to stand in the way of their expressing an opinion). This could lead to certain countries, e.g. France, being downgraded relative to those 'willing to have a go.'

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(41) Ibid., para 8.37
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Postscript

Revised mathematics proposals have now been published by the National Curriculum Council. Some changes have been made to content and, in particular, Profile Component 3 has been abandoned. The changes do not, however, meet the fundamental objections of this paper. Some, for example the inclusion of 'long division' at level 5, reinforce its opposition to the underlying model. Long division is an admirable way of gaining familiarity with numbers, of learning to estimate (*how many times will . . . go into . . .?*), and of preparing for future work in polynomial algebra. As such it should be attempted by all in the top third of the ability range and early in their mathematical careers (i.e. it is not suitable for the higher TGAT levels). Yet to ask below-average pupils to devote much time in their last two years of schooling to acquiring fluency in its use hardly seems wise. Again we see the drawbacks of a policy which enforces the same curriculum on all, irrespective of age and ability.

The major significance of the dropping of Profile Component 3 does not concern its presence or absence in the National Curriculum. It is reported that 80% of responses to the original proposal were in favour of its retention. Certainly, all those from the professional associations argued for it to be kept. As has happened before, however, professional opinion has been sought and then set aside. Such actions only depress morale within the educational system yet deeper.

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