CENTRE FOR POLICY STUDIES

# POLICY CHALLENGE

*1996 Education Summer Series*

# Standards of Arithmetic

## *How to correct the decline*

JOHN MARKS

CPS

*The 1996 Education Summer Series*

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1996

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# SUMMARY AND RECOMMENDATIONS

**Summary**

Standards of mathematics in Britain's schools are low: a major cause is likely lie in the way in which arithmetics is taught in primary schools.

Standards of mathematics in competitor countries are higher. Only the top quarter of British sixteen year-olds attain the same standards in mathematics as the average Japanese fifteen year-old; when asked to work out 1/2 x 4/5, 72% of German thirteen year-olds were able to answer correctly; in Britain only 20% could answer the same question.

Arithmetic is the foundation stone upon which later mathematics is built.

Low standards in mathematics can be attributed to the failure of British primary schools to teach arithmetic properly. In 1991, fewer than 15% of seven year-olds were able to do simple multiplication sums such as 5 x 5.

Variations in the standards of mathematics in different countries are associated with the differences in curricula, teaching methods (including the use of textbooks) and classroom organisation.

**Recommendations**

The National Curriculum should be reformed so that much more emphasis is placed on Arithmetic in Key Stages 1 & 2. Calculators should not be allowed in primary schools.

Standardised tests on accuracy in arithmetic should be administered and the results for each school published.

OFSTED should ensure that all primary school children are taught arithmetic using traditional teaching methods and practices similar to those found on the Continent.

CHAPTER 1

# STANDARDS IN THIS COUNTRY

CONCERN ABOUT STANDARDS OF arithmetic in British schools is not new. As long ago as 1977, a survey of 8,000 sixteen year-olds by the Institute of Mathematics[1] showed significant weaknesses in basic computational skills with, for example, 25% of London's sixteen year-old schoolchildren unable to multiply 6 by 79 even with a pencil and paper.

There were also many disturbing findings in the Second International Mathematics Study (SIMS) in 1981. These findings have been succinctly summarised and discussed by Professor Geoffrey Howson who concluded that, for English pupils:

> ...the results at 13+ were deeply disappointing and disturbing...
>
> Overall we were ranked 11th in attainment, being well behind Japan, Hungary and France...
>
> Our results in Arithmetic and Algebra were...particularly weak...

Moreover, in comparison with a similar international study in 1964 using many of the same questions:

> ...our results dropped from 52% to 44% and we were the only country to have recorded lower levels of attainment in all the four divisions: Arithmetic, Algebra, Geometry and Measurement, and Statistics.[2]

Such findings have been confirmed in subsequent surveys of eleven and fifteen year-

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1 'A Pilot Test of Basic Numeracy of Fourth and Fifth Year Secondary School Pupils', *Bulletin of the Institute of Mathematics and its Applications*, Vol 14, No 4, April 1978, pp 83-6; reprinted in C Cox and J Marks (eds.), *The Right to Learn,* Centre for Policy Studies, 1982.

2 G Howson, *Maths Problem: Can more pupils reach higher standards?,* Centre for Policy Studies, 1989.

olds by the Assessment of Performance Unit which even showed a decline in basic computational skills during the 1980s in England and Wales (but not in Northern Ireland).

Evidence concerning primary school standards in mathematics was also summarised by the 'three wise men' appointed by the Secretary of State for Education in 1991 to review primary education in Britain.[3] They concluded that there had been 'a worrying deterioration in important aspects of numeracy' over the last two decades.[4]

More recent evidence also suggests that low standards in mathematics may, in part, have their origins in primary schools. The first results of the National Curriculum tests for British seven year-olds for 1991 showed particularly disappointing results for mathematics. The original expectation had been that about one pupil in six would reach the highest level of attainment; this standard was in fact reached by only one in 20. Only one in seven could do simple sums such as 5 x 5 or answer a series of questions, including the cost of three 50p loaves of bread, or the number of 25p packets of crisps, which could be bought for £1.50. As few as 56% were able to do a series of sums such as 5 + 4 or work out, for example, how much change is left from 20p after buying a 10p doughnut and an 8p currant bun.[5]

Much more recently, in November 1995, a substantial report[6] – produced by a group chaired by Professor Howson and published by the London Mathematical Society – on standards at A-level and entry to university showed problems with:

- arithmetical operations;
- knowledge of and facility in algebra;
- lack of awareness of the concept and nature of proof.

In other words this report identifies weaknesses in the traditional subjects of arithmetic, algebra and geometry.

Finally, recent reports[7] concerning the results for English thirteen year-olds in

[3] R Alexander, J Rose and C Woodhead, *Curriculum Organisation and Classroom Practice in Primary Schools,* DES, 1992; see Section III, Standards of Achievement in Primary Schools, paragraphs 24-50.

[4] Ibid., paragraph 35; the authors also found 'some evidence of downward trends in important aspects of literacy' (paragraph 50).

[5] Analysis of the 1995 National Curriculum results for eleven year-olds also shows that poor standards in mathematics are widespread; see J Marks, *Standards of English and Maths in Primary Schools for 1995,* Social Market Foundation, 1996.

[6] *Tackling the Mathematics Problem,* The London Mathematical Society, 1995.

[7] J O'Leary, 'What's wrong with our maths?', *The Times,* 26 July 1996.





Mathematics in 1995 in the Third International Mathematics and Science Study (TIMSS) claim that, when it is published in November 1996, the full international study will show:

> ...England sliding down the world league at an alarming rate: from 3% above the international average six years ago to 3% below now.
>
> English thirteen year-olds are bottom of a subset of nine comparable countries out of the 41 taking part...only the Americans tested for the study answered as few questions correctly as the English in Year 8 of the school system, and even the Americans were marginally ahead in Year 9. Table-topping Singapore was 26% percentage points ahead of England in both years.

The implications for primary schools are mainly related to weaknesses in arithmetic and in particular in the training required to develop manipulative facility in arithmetic using the four basic rules of addition, subtraction, multiplication and division, and in the use of fractions and ratios.

This facility is vital for two reasons – first because it is needed in much of the simple mathematics involved in practical or applied situations and problems and secondly because the types of manipulations that are involved in arithmetic are exactly similar to those which will be needed later in the development of facility in algebra; elementary algebra is best regarded as a kind of generalised arithmetic.

CHAPTER 2

# STANDARDS IN OTHER COUNTRIES

DURING THE 1980S EVIDENCE also accumulated about the low standards of British schoolchildren in comparison with competitor nations. In 1985 the National Institute for Economic and Social Research (NIESR) concluded that the standards achieved in mathematics in West German and English schools were roughly comparable for those at the *top* of the ability range but that the *average* level of attainment for all pupils is appreciably higher in Germany.[8] In particular:

> Attainments in mathematics *by those in the lower half of the ability-range in England appear to lag by the equivalent of about two years' schooling behind the corresponding section of pupils in Germany* (p. 42; emphasis in original).

A similar study of Japanese schooling has also shown dramatic differences between Japanese and English pupils.[9] The average Japanese fifteen year-old is better educated in mathematics than the top 25% of British sixteen year-olds who pass at GCSE higher grades (A – C). The study concludes:

> The broad cross-section of school-leavers in Japan is...educated to a significantly higher level than in England; it is as if good Grammar School standards were attained by the school leaver of average ability.[10]

International studies involving many countries confirm these findings. For example, in the International Assessment of Educational Progress (IAEP) study in 1991, English pupils' average score for arithmetical operations at the age of nine was below that for 12 of the 14

[8] S J Prais & K Wagner, 'Schooling Standards in Britain and West Germany', *National Institute of Economic and Social Research*, 1985.

[9] S J Prais, 'Education and Productivity: Comparisons of Japanese and English Schooling and Vocational Preparation', *National Institute Economic Review*, No 119, February 1987, pp 40-56.

[10] Ibid., pp 52, 45.





participating countries and only just above that for the bottom country, Slovenia.[11]

Moreover, early data from the Exeter-Kassel project[12] shows that German thirteen year-olds performed significantly better than English pupils, especially in Number (as Arithmetic is called in the National Curriculum). The following table shows some of the differences on relatively straightforward questions.

| | Percentage answering correctly | |
|---|---|---|
| | **England** | **Germany** |
| 900/30 = ? | 56 | 93 |
| 1/2 – 1/3 = ? | 21 | 61 |
| 1/2 x 4/5 = ? | 20 | 72 |
| Simplify 2x + 5x | 36 | 66 |

Preliminary results from the second year's work on this project also show that German pupils make greater progress than English pupils and that results in Number from German Hauptschule (Secondary Modern) pupils are better than those for *all* English pupils.

The exacting nature of the primary school curriculum in France can also be seen from the comments[13] of Her Majesty's Inspectorate (HMI) on the standardised Mathematics tests taken by French eight year-olds on successive days in one week at the beginning of the school year:

> The language used in the mathematics tests is precise and mathematical and the level of reading competence quite demanding, e.g. matrix, perpendicular, interior, exterior, segment. In the first two tests pupils are required to recognise shapes, patterns and sequences; to solve problems posed verbally; tell the time; measure accurately and make comparisons; and read and interpret from tables of information. The third and fourth tests require the ability to add and subtract hundreds, tens and units and multiply by a single figure. Linear computation, using brackets, is also included. Mental arithmetic, sequencing and recognition of number patterns follow. The final test requires the ability to answer questions from pictorially given information, to identify an operation from a table of information, and to explain the answer given.

[11] A E Lapointe, N A Mead & J M Askew, *Learning Mathematics,* Educational Testing Service, Princeton, 1992; for further discussion see H Bierhoff & S J Prais, 'Schooling as Preparation for Life and Work in Switzerland and Britain', *NIESR*, 1995.

[12] D Burghes & W Blum, 'The Exeter-Kassel Comparative Project: A Review of Year 1 and Year 2 Results' in *Mathematics Education,* The Gatsby Foundation, November 1995.

[13] *Aspects of Primary Education in France*, HMI, 1991.

CHAPTER 3

# DIFFERENCES BETWEEN COUNTRIES IN THE TEACHING OF PRIMARY SCHOOL MATHEMATICS

Variations in the standards of mathematics in different countries are associated with differences in curricula, teaching methods and school organisation.

**Differences in Curricula**

Since its introduction in the 1988 Education Reform Act, the National Curriculum in England has insisted that:

> ...all pupils aged five to sixteen, irrespective of their ability, attainments or vocational intentions should be required to study equally all the designated branches of mathematics (originally 14 Attainment Targets were distinguished, subsequently combined to five). There is scant recognition in our National Curriculum, as there still is in mainland Europe, that at primary school ages it is better to concentrate on arithmetic; and even at secondary school ages, those pupils who have difficulty in mathematics have more to gain by concentrating on arithmetic...in contrast to our National Curriculum requirements, there are no investigations, and negligible data handling, statistics or probability.[14]

Moreover, in mainland Europe, the official curriculum is usually specified separately for each year of schooling, and not for groups of years – such as for all four of the primary years from seven to eleven or for all five of the secondary years together, as under the latest Dearing reforms.[15] Similar close specification is the rule in Japan where the Ministry of Education (Monbusho) produces mandatory national curricula for mathematics, as for all other subjects, separately for each year group for all ages from six to eighteen. Moreover the courses of study:

> ...specify not only the nature and levels of topics to be covered, but also the

[14] S J Prais, 'Improving School Mathematics in Practice' in *Mathematics Education,* The Gatsby Foundation, November 1995, pp 3, 7.

[15] Ibid., p 7.





> number of hours to be spent in each grade on each subject.[16]

The relatively poor mathematical performance of English pupils in international studies such as SIMS[17] has sometimes been attributed to the wider range of the English curriculum; poorer performance in Number is supposed to be compensated by greater breadth of knowledge.

Such curricular differences were one of the arguments which led to the initial decision that this country should not take part in the Third International Mathematics and Science Study (TIMSS). The other kind of argument used was that the kind of objective written tests involved in such international studies were inappropriate to the new styles of Mathematics teaching and assessment which were increasingly being adopted in this country. This decision has now been reversed for nine and thirteen year-olds, but not for eighteen year-olds; the results should become available sometime in 1996.

A further example of these curricular differences is given by the HMI report on primary schools in France.[18] HMI found that, in teaching mathematics, French primary schoolteachers concentrated mainly on Number and geometry.

> In both areas of the subject the [French] children, particularly those of average ability, achieved good standards. Mental arithmetic and written computational skills were especially good...

Throughout, in French schools:

> There was a strong emphasis on understanding number operations, making use of methods of calculation, and on shape and space. Broadly speaking in these areas of the curriculum, the standards at the end of Key Stage Two [i.e. at the age of eleven] were often better than those achieved in our own schools, particularly for the pupils in the middle range of ability.

**Whole Class Teaching**

There are considerable problems in England:

> ...because of widespread individualised teaching methods which require children to learn by themselves for a great proportion of the time, English

[16] J Whitburn, 'The Teaching of Mathematics in Japan: an English perspective', in *Oxford Review of Education*, Vol 21, No 3, 1995, p 352.

[17] See page 1 above for details of these results.

[18] *Aspects of Primary Education in France*, HMI, 1991.

> Teachers spend most of their time in class dealing with individual pupils; each pupil therefore has very limited contact time with the teacher, and consequently benefits from no more than a few minutes of direct teaching in each lesson.[19]

This has been confirmed by recent OFSTED reports. For example:

> Pupils frequently learn new topics largely on their own from written texts.
>
> ...too many pupils spend a lot of time working individually through texts, with little direction from, or interaction with, the teacher.[20]
>
> ...the teacher's time is spent...insufficiently in direct teaching with all pupils.[21]

Similar differences are apparent in comparisons with Japanese schools:

> A major difference...is the predominance of whole-class teaching in Japan in contrast to the child-centred approach widely used in England. Encouraging each child to progress at his or her own pace seems likely to increase the range of attainment with the slower pupils falling further and further behind as their education accumulates. In contrast the Japanese approach teaches the great importance of keeping the whole class together...[22]

Moreover in French primary schools HMI reported that:[23]

> ...similar tasks were given to the whole class at the same time. Each lesson always included a well-defined beginning and a sequence of small steps towards a pre-determined goal. Exposition by the teacher alternated with tasks of relatively short duration which enabled the children to practise what they had heard explained at each stage.
>
> Whole-class teaching and close direction of lessons by the teacher resulted in

[19] H Bierhoff, 'Laying the Foundations of Numeracy: A comparison of primary school textbooks in Britain, Germany & Switzerland', *NIESR*, January 1966, p 4.
[20] *Mathematics: Key Stages 1, 2, 3 and 4, Fourth Year, 1992-93*, OFSTED, 1993, paras. 14, 15.
[21] *Review 1993/94*, OFSTED, paras. 12, 14.
[22] Whitburn, op. cit., pp 358-9.
[23] *Aspects of Primary Education in France*, HMI, 1991.





> some significant gains. The level of teacher expectation was high, the activities proceeded at a brisk pace, and there was usually a clear progression within the lessons themselves. The quality of exposition and the use of questioning by the teachers were invariably very good, enabling them to hold the attention of most of the class for long periods at a time.

However, HMI did find one exception to the general pattern of whole class teaching. At one school:

> ...the emphasis was on individualised learning using published workbooks... which...enabled the children to work at their own level and to mark their own work.
>
> ...this form of organisation relied too heavily on the published workbooks, reducing the scope for the use of other teaching strategies and making it difficult for the teachers to introduce new material of their own. Consequently a high proportion of the tasks were concerned more with consolidation than with the learning of new concepts or skills and the pace of work was much slower than in the lessons with greater teacher input.

**Use of Text Books**

The most recent comparison of English and German Mathematics text books for eight year-olds has been conducted by Helvia Bierhoff at the National Institute for Economic and Social Research. This study showed that the most important differences were that:

> 1. Mental arithmetic is given precedence over vertical written methods on the Continent. This reflects the widely accepted Continental view that mental calculation promotes pupils' conceptualisation of number; and that mathematics provides a unique subject for training the *minds* of pupils in extended chains of reasoning.
>
> 2. The Continental approach to teaching two-digit numbers emphasises working with whole numbers taken as single concepts. The English approach concentrates on 'dissecting' numbers according to the place-value of their constituent digits. It seems likely (from the kind of typical errors made by pupils) that English pupils' slow progress in extending their arithmetical competence is related to the underemphasis on whole-numbers taken as single concepts and an overemphasis on the place-value of each digit.

3. Pupils' introduction to the new number-range beyond 20 and up to 100 is significantly less thorough in England than on the Continent.

4. Exercises in Continental textbooks are designed to support the development of optimal mental calculating-strategies. In England, children are expected to develop their 'own methods', making extensive use of concrete material. In practice, this encourages English pupils to continue for too long counting one by one, using their fingers (counters or bottle tops, etc.). Counting as a strategy to solve arithmetical problems is regarded on the Continent as a proper – but early and 'primitive' – stage in the development of mental processes, to be overtaken as soon as possible by mental methods. Continued for too long, it impedes the development of mental calculating-strategies.

5. The introduction of pocket calculators at primary school is regarded by Continental educators as carrying risks of retarding that mental development, and thus vitiating a prime objective of mathematical education.

6. Consolidation of foundations is given more weight and space in Continental than English textbooks. Three times as many examples for pupils to work on are provided in Continental texts. Thorough consolidation is considered important for pupils across the whole attainment-range.

7. Larger continuous sections of time are spent on each topic on the Continent before moving on to the next. This is reflected in textbooks where, typically, there is an average of 12 pages on each topic in Continental textbooks, compared with an average of six pages in English textbooks. Taking into account that Continental textbooks contain about three times as many exercises on each page as English textbooks, Continental pupils have available some six times as many exercises on a topic as English pupils before moving on to the next topic.

8. Progression is less distinct in English than in Continental textbooks. In English textbooks, there is recurring repetition of simple concepts; and more difficult concepts are introduced before pupils are assumed to have mastered underlying simpler concepts.[24]

[24] Bierhoff, op. cit., pp 40-42.





This study also makes the point that the greater role of whole-class teaching and the provision of more detailed teachers guides to help teachers plan their teaching are aspects of the use of text books which may enhance their importance and effectiveness in Germany as compared with Britain.

> Another important difference may be that in Germany, and many other countries too, textbooks have to be approved by the appropriate Ministry of Education before they can be used in schools.

This is certainly true in Japan where Monbusho approves a small number of textbooks for each subject at each grade and each child is provided with a copy to keep. Monbusho also approves extensive Teachers Guides which contain:

> ...the answers and solutions to the questions set in the pupil's book; points of explanation which should be covered by the teacher, together with the questions to be asked of the class, and examples to be demonstrated on the board. The use of such 'Teachers Guides', containing clear explanation, exposition and worked examples suggests that even with poor teachers, pupils will receive adequate mathematical instruction. With a copy of a Teachers Guide, it is not difficult for a non-specialist in mathematics to give a competent lesson of an acceptable standard, whereas many teachers of mathematics in England spend many hours preparing their own teaching materials, work sheets and practical investigations.[25]

**Use of Calculators**

As has been mentioned above, calculators are not used in German primary schools. Nor are they used until a fairly late stage in German secondary schools. Similarly in Japan:

> ...virtually no use is made of them as a mathematical tool. Until 1993, calculators were not used at all in secondary schools, either in lessons or in examinations.

Moreover they are not used in Japanese primary schools either. However:

> Japanese teaching methods...do not oppose the use of computational aids: until recent years the *soroban* (or abacus) was used by all elementary school children to facilitate basic arithmetic. Many teachers to whom I spoke

[25] Whitburn, op. cit., p 353, 356.

This emphasis was later picked up and incorporated in the National Curriculum for Mathematics and became a major part of what was required by that Curriculum. In the 1991 version of the National Curriculum, Attainment Target 1 – Using and Applying Mathematics became primarily a forcing ground for the development of 'Investigative Mathematics' of a particularly time-wasting and unconstructive kind. For example, the National Curriculum in England requires 11-year-old pupils to:

> ...test the validity of statements such as...it is harder to get a six on a dice than a one;

and to:

> ...observe from data they have collected that woodlice prefer dark, damp conditions because more of them are found under stones, damp rubbish, etc.[30]

According to researchers from the National Institute of Economic and Social Research:

> Nothing corresponding to such activities was observed in Continental schools visited by the National Institute teams, nor was it found in their National Curricula or textbooks.[31]

Moreover, under the 1991 National Curriculum:

> A fifth of the total marks in mathematics was required...to be devoted to this investigative branch of the subject (misleadingly called 'Using and Applying Mathematics'), just as a fifth is to be devoted equally to each of the other four topics distinguished by the National Curriculum: Arithmetic (called Number in the National Curriculum), Algebra, Geometry (called Shape and Space) and Handling Data. It may seem surprising that the weightings devoted to these five topics should be equal; and even more surprising that the weightings are equal throughout the age range and throughout the attainment range. It contrasts with widely accepted teaching practice that a child should begin predominantly with arithmetic; and that even at secondary school for those who have difficulty in mathematics, the mastery

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in D O'Keeffe (ed.), *The Wayward Curriculum,* Social Affairs Unit, 1986, pp 23-29.

30 These examples are quoted from the *National Curriculum in Mathematics,* HMSO, 1991.

31 S J Prais, *Productivity, Education & Training: An International Perspective,* Cambridge University Press, 1995, p 88.

of arithmetical topics is to be given priority. That remains the approach elsewhere, including the Continental schools we visited.[32]

During the 1993-4 Dearing Review of the National Curriculum, it was strongly argued that fostering such 'Investigative Mathematics' was unhelpful and was not what most people understood by using and applying mathematics. What was needed, it was suggested, was to incorporate the requisite applications of mathematics within the programmes of study dealing with the mathematical concepts themselves. Thus arithmetic would include some practical applications of arithmetic and geometry. This is how most National Curricula in other countries with higher standards in arithmetic incorporate the applications of mathematics. Nevertheless, despite such recommendations being strongly put forward, nothing was done to modify the statutory curriculum in this respect.

In this context, it is of interest that in the TIMSS study mentioned in Chapter 2, in addition to the main comparative studies at the ages of nine and thirteen, which mainly use standardised written tests of Mathematics, there are also two further studies. One is for nine year-olds, the other for thirteen year-olds. They both involve more practical or investigative types of mathematics of the kind which many, in this country, have advocated should be the staple diet of most of our school children. Originally it was intended and agreed that England should take part in these supplementary tasks for both nine year-olds and thirteen year-olds. More recently, however, it was decided that nine year-olds in this country should now withdraw from these tasks. The reasons given for dropping out at this late stage were that, in the pilot studies, the nine year-old pupils in the sample schools were unable to read the instructions sufficiently well, were unable to follow the instructions through so as to successfully complete the tasks even when they could read them, and were unable to concentrate on a task for a sufficient length of time. This is surely an extraordinary indictment of the general failure of our primary schools.

[32] Ibid., pp 88-89.



CHAPTER 4

# WHAT CAN AND SHOULD BE DONE

WHAT CAN AND SHOULD be done follows directly from the arguments above.

**The National Curriculum**

The National Curriculum should be brought into line with National Curricula in other countries by:

- putting *much* more emphasis on arithmetic in Key Stages 1 & 2;
- cutting out the separate sections on Data Handling and Using and Applying Mathematics,
- ceasing to require the use of calculators in primary schools.

Moreover all National Curriculum tests should be done without the use of calculators.[33]

**Accuracy in Testing**

Second the National Curriculum Tests in Mathematics at ages seven and eleven should focus primarily on accuracy in basic arithmetic, both mental and written. Such tests should be produced in standardised form so as to give as much discrimination and accurate information as possible and their results should be published forthwith school by school. Once again, this should be done at once even if it means changing the current testing framework, including the discredited 8-level (formerly 10-level) scale.

[33] This should be done at once despite the existing moratorium on further change until the year 2000. It is ridiculous that we should continue to require teachers to follow by law practices which are so detrimental to the sound learning of the foundations of mathematics, particularly when all that is required is a change in Statutory Regulations, which can be passed very quickly given all-party agreement, rather than primary legislation.

**Style of Teaching**

Significant changes in teaching styles, classroom organisation are necessary. The use of simple teaching aids such as blackboards and textbooks along the lines which have proved to be so successful in other countries should be introduced. The motivation for such changes should be sought in greater knowledge of what can be achieved – perhaps by the use of videos and exchange visits.[34] It would also be helpful if more use could be made of what has been learnt about adopting continental teaching practices in English schools in a continuing project in the London Borough of Barking and Dagenham.[35]

**Textbooks**

Particular efforts are needed in order to produce much better textbooks and Teachers Guides; translating and using existing continental books would be an interesting experiment.[36]

**The Role of OFSTED**

It should be the responsibility of OFSTED to ensure that such changes are implemented. In particular, the 10 new centres for the teaching of numeracy which are being financed by the Government over the next five years should be used to introduce such initiatives. It is essential that these centres should disseminate the kind of practices described above and not perpetuate the failed methods of the last two decades.

**The importance of arithmetic**

Arithmetic is important because, if learnt at the right early age, it provides the key to a whole host of practical applications in all walks of life.

And it is even more important because the precision of arithmetic, if properly taught, performs two vital intellectual functions. It provides a model for the later introduction of algebra, itself a kind of generalised arithmetic.[37] And it provides a first taste of, and schooling in, a discipline in which the results are *absolutely exact* and precise. Such an understanding is vital to the later appreciation of the importance of

[34] For further detail, see S J Prais *Improving School Mathematics in Practice* in *Mathematics Education,* The Gatsby Foundation, November 1995, especially pp 8-9.

[35] G Last, *The Gatsby Primary Mathematics Project: A demonstration project in the London Borough of Barking and Dagenham* in *Mathematics Education,* The Gatsby Foundation, November 1995.

[36] Bierhoff, op. cit., p 42.

[37] Algebra is one of the glories of the development of mathematics in the work of Viete and Descartes in the 16th and 17th centuries. It later provided the basis of the differential and integral calculus. For further information, see J Marks, *Science and the Making of the Modern World,* Heinemann Educational, 1983, Chapters 3.3 & 3.4.





exact proof in mathematics or in logic. As one experienced teacher of mathematics has put it:

> In mathematics, the '=' symbol conveys a *moral* message...once the moral imperative of the '=' symbol is lost, mathematics becomes no more than an experimentally based bag of tricks.[38]

So let us have the courage to call it arithmetic and to teach it in ways which will lay the right foundations for the later stages of education and provide a proper logical, and practical, foundation for the minds of the next generation.[39]

[38] T Gardiner, *Observed effects of recent changes in English school mathematics on those entering universities at age 18,* University of Birmingham, 1994.

[39] For a broader discussion relating to mathematics in secondary and vocational as well as primary education, see S J Prais, *Productivity, Education & Training: An International Perspective,* Cambridge University Press, 1995, pp 75-90.