Centre for Policy Studies

The Myth of Inflation Targeting

The largest policy failure of our generation

MAURICE SAATCHI

THE AUTHOR

Maurice Saatchi is Chairman of the Centre for Policy Studies. He was the co-founder of Saatchi & Saatchi and is now a partner in M&C Saatchi. He is a Governor of the London School of Economics. He is the author of *The War of Independence* (1999); *The Bad Samaritan* (2000); *Poor People! Stop Paying Tax!* (2001); *The Science of Politics* (2001); *In Praise of Ideology* (2006); *The Sleeping Beauty of America* (2007); and *Enemy of the People* (2008). He was elevated to the peerage in 1996. He served on the Conservative Front Bench in the House of Lords as Shadow Minister for the Treasury and the Cabinet Office between 1999 and 2003. He was Co-Chairman of the Conservative Party from 2003 to 2005.

ISBN 978-1-905389-98-8

Printed by 4 Print, 138 Molesey Avenue, Surrey

CONTENTS

SUMMARY

- When things go wrong, it is a human instinct to attribute blame.
- Today, many people are routinely blamed for this economic crisis – investment bankers, for greed; central bankers, for being “behind the curve”; regulators, for being “asleep at the wheel”; credit rating agencies, for too many Triple A ratings; even the public themselves, for foolishly borrowing too much.
- What made all these people reckless at the same time?
- The record seems to show that they were all just victims. Their mistake was to believe what they were told. They were lulled into a false sense of security by an idea – that if policy makers could maintain low inflation (and more important, low inflation expectations), then all good things would follow – growth, employment, prosperity, stability.
- Unfortunately, the idea turned out to be a myth – the largest policy failure of our generation.

- The Myth of Inflation Targeting created the illusion of the new Jerusalem, the new paradigm – the end of the economic cycle.

- But lack of clarity about the distinction between the necessity and sufficiency of inflation targets has been responsible for a misunderstanding of epic proportions.

- By first creating the false impression that low inflation meant financial stability, and then measuring the wrong kind of inflation, the inflation targeting policy encouraged the view that it was safe to borrow, safe to invest.

- The Myth led bankers to lend more, traders to risk more, homeowners to borrow more, regulators to relax more, and politicians to boast more – about the end of boom and bust.

- When the Myth collapsed, it took all of us down with it – academics and auditors, bankers and bakers, economists and electricians. We all went into the dark.

- So this pamphlet has a definite purpose:

  Never again

1. INTRODUCTION

The first chapter of this pamphlet deals with the error in the Bank of England's remit of obliging the Bank to focus only on inflation.

The second chapter addresses how that error was compounded by obliging the Bank to focus only on one kind of inflation – as it happens, the wrong kind.

And the third chapter deals with the consequences – how everyone was duped by this policy failure into an inappropriate sense of self-confidence, including bankers and their auditors. This chapter seeks to impose a duty on auditors to investigate and report on Banks' exposure to "insured" liabilities held "off-balance sheet".

These proposals will not abolish the economic cycle, as was claimed by the Myth of Inflation Targeting. But, taken together, if these changes were enacted in UK law, Britain would never again have an economic crisis caused by a banking crisis caused by debts that went unseen by auditors, regulators and central banks.

This pamphlet does not seek to apportion blame for the crisis. Failure of free markets. Failure of regulation. Failure of the FSA. Or the Bank of England. Or the Treasury. Or the whole tripartite system. Failure of the UK. Or failure of the US. Or of the whole world. The pamphlet is neutral on all counts.

It has only one modest aim. To help whichever party is in Government to be wary of the idea that an economic disaster can not occur during a period of low inflation.

2. TARGETING ONLY INFLATION

Shakespeare taught us that human beings can have a fatal flaw. So too can legislation.

The Bank of England Act 1998 has a fatal flaw: it has three words too many.

All of us know the importance of language. Parliamentarians are often mocked for endlessly debating tiny amendments to legislation, such as:

> “Delete ‘a’. Insert ‘the’ ”

But in this case, three words changed history.

These days, Bank of England officials are routinely condemned for allowing British citizens to suffer the longest and sharpest recession of all major economies.

But, in fact, the top officials of the Bank of England were like top generals given the wrong orders. The fault lies not with them, but with the legislation that created them. The Myth of Inflation Targeting left our Central Bank with no function other than to deliver low inflation expectations.

This pamphlet is based on the unremarkable proposition that officials of the Bank of England have sufficient wisdom and breadth of vision to see the whole economic picture.

They should not be forced to wear legislative blinkers which blind them to how an economic disaster can arise during a period of low inflation.

The Bank of England Act 1998 was the iconic Act of Parliament which gave the Bank of England its independence. Clause 11 in Part 2 of the Act defined the role of the Bank. It reads:

> "In relation to monetary policy, the objectives of the Bank of England shall be –
>
> a) to maintain price stability, and
>
> b) **subject to that**, to support the economic policy of Her Majesty's Government, including its objectives for growth and employment."

The Bank of England was ordered to concentrate on inflation above all else, so that when the crunch came its top officials were looking the other way.

During the passage of the Act in the House of Lords, there was much discussion about those three words, "subject to that".

Why not, "having regard to"? Or, "taking account of"? Why were all other considerations to be subordinate to controlling inflation?

The economic orthodoxy which underpinned the Act was based on the seminal work of Professors Paish and Phillips at the London School of Economics.

The ‘Phillips Curve’ showed that high inflation was incompatible with high employment and high growth.

The mechanism of the causal relationship between them was ‘Wage Push’. According to the theory, noting higher prices, unions would press for higher wages, and employers would lay off workers to compensate. This was the dreaded wage/price spiral of the 1970s. Policy makers concluded, and legislators concurred, that inflation must be curbed at all costs.

The winning argument at the time of the Act was that the relegation of the Government’s *growth and employment objectives* would prevent the manipulation of economic activity by unscrupulous politicians in search of votes. And control of inflation was, in any case, the best guarantor of growth and employment.

The British psyche had been scarred by the “Stop/Go” cycle of the 1970’s. A boom. Followed by inflation. Followed by severe measures to control inflation. A bust.

The lesson seemed obvious – if the monster of inflation could be tamed that would be the end of the economic cycle as we know it.

On 12 November 2008, in reply to a question in the House of Lords about the remit of the Bank of England by former Chief Secretary of the Treasury, Lord Barnett, the UK Government was still clinging to the wreckage of that theory.

Lord Myners spoke for the Government. He said it was quite satisfied with the remit of the Bank of England. It was impeccable. In fact, it was so perfect that it had been responsible for the fine performance of the economy under his Government. He said flat out that:

> "The remit should not be amended in any way."
> [Official Report, 12/11/08; col.655]

And he gave his reasons. The remit had delivered "unparalleled economic growth and low inflation."

He explained how it had achieved this. He said that stable prices and economic growth are "not in conflict; one is a pre-condition of the other."

And he went on to say that low inflation was:

> "an essential precondition of growth and prosperity."

In other words, in November 2008, the UK Government still thought that low inflation would deliver economic growth, and financial stability.

Unfortunately, as we all know, it has not worked quite like that.

A month later, the UK Government had changed its mind. It had found a flaw in the Bank of England's remit. On 4 December, it published the Banking Bill 2008 to amend it.

Section 2 A (1) of Clause 228 of the Government's Banking Bill would give the Bank of England a second new objective:

> "to contribute to protecting and enhancing the financial stability of the United Kingdom"

So in this little noticed revolution in economic management, the tectonic plates of the Bank of England were shifted from the sufficiency of a single remit to the necessity for a dual mandate.

What could have led the UK Government to such a change of mind about the remit of the Bank of England?

The Government belatedly agreed that while low inflation may be a necessary condition for financial stability, it is certainly not a sufficient condition.

That fine theory, to which the Government was still advocating November 2008, had already met its Waterloo in October 2008, when we discovered, according to the Deputy Governor of the Bank of England, that,

> "the largest financial crisis in human history"

had arisen during a period of low inflation.

By focusing only on inflation, as three words in Clause II of the Bank of England Act 1998 directed it to do, the Bank of England was blindfolded to the disaster for economic growth and financial stability that could occur in a low inflation environment.

Never again

.

3. TARGETING THE WRONG KIND OF INFLATION

The previous chapter explains the error in the Bank of England's remit of making the Bank focus on inflation above all else. This chapter deals with how that error was compounded by obliging the Bank to focus only on one *kind* of inflation – the wrong kind.

At the time of the Bank of England Act, the Retail Price Index inflation target (RPI) included housing payments. However, in 2002 the CPI (Consumer Price Index) was introduced, which excluded housing costs. Whereas the Bank's CPI keeps a close eye on inflation in the price of a packet of peas and a bar of chocolate, it overlooks the very aspects of inflation that caused this crisis – all the debt, housing, mortgage ingredients of our present misfortune.

While asset prices rose and fell by 50%, the Bank of England was directed by the Act of Parliament which created it to look the other way – to painstakingly examine a rate of inflation which barely moved at all.

But while 'inflation' stayed between a narrow range, other rates of inflation changed dramatically.

INFLATION OF DEBT OWED BY THE AVERAGE UNITED KINGDOM HOUSEHOLD

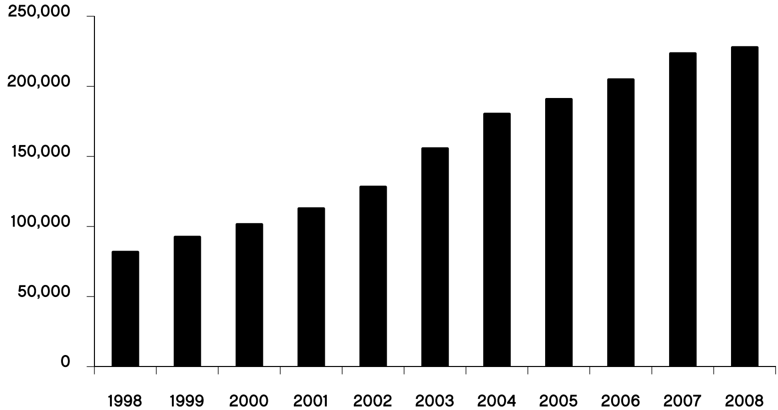
Year	Total financial liabilities of households and non-profit institutions serving households (£ millions)	Total number of households (millions)	Average Household's Financial Liabilities (£000s)
1996	550,108	23.68	23,231
1997	586,659	23.8	24,650
1998	625,134	23.98	26,069
1999	675,599	24.08	28,056
2000	734,790	24.28	30,263
2001	810,665	24.43	33,183
2002	923,144	24.62	37,496
2003	1,046,913	24.68	42,419
2004	1,172,032	24.74	47,374
2005	1,249,393	24.83	50,318
2006	1,405,756	24.88	56,501

Source: www.statistics.gov.uk/statbase/product.asp?vlnk=1904

For the last five years, 'debt inflation' was on average 11.25% a year, nearly five times the Bank of England CPI inflation target. But where is 'debt inflation' in the CPI?

Not included.

INFLATION OF ASSET VALUE: AVERAGE HOUSE PRICES



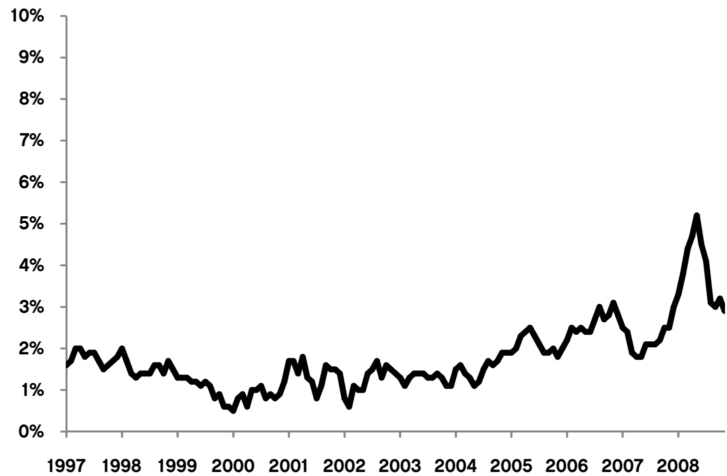
Source: http://www.communities.gov.uk/housing/housingresearch/housing statistics/housingstatisticsby/housingmarket/livetables

During the same period, the inflation rate of one particular asset class, property, was 13% p.a., five times the Bank CPI target.

Where is that in the CPI? Not included.

What about the cost of acquiring and holding these property assets; i.e. mortgage interest? Not included.

CPI INFLATION AS MEASURED BY THE BANK OF ENGLAND



Source: http://www.statistics.gov.uk/statbase/product.asp?vlnk=868

CPI inflation barely moved. It was irrelevant. It neither registered the huge increase in asset price inflation, nor the huge collapse in asset price inflation.

The Bank's measure of inflation was stuck in the past. The Bank's CPI measure of 'inflation' is out of touch because the world has changed. Millions of people had become investors in a booming new asset class. They were home owners. We are used to lamenting the scarcity of private investors in the stock market. But people did not stop investing. They just found something better to invest in – property.

"I borrow money.

I buy an asset.

The price goes up.

> I exit the asset.
>
> I repay the loan.
>
> I keep the profit."

This was the joy of debt, as also understood by the masters of private equity.

The great British public had been given specific assurances that:

> "Central Banks had secured low inflation.
>
> Central Banks had managed low inflation expectations.
>
> Central Banks had achieved predictably low inflation.
>
> Central Banks had said that meant prosperity and stability."

There was nothing to worry about.

But the great ones did not consider how a change in the inflation rate of a certain asset class could bring about a dramatic collapse in the economy, notwithstanding "low inflation". As the Chairman of the US Federal Reserve, Ben Bernanke, put it, there is no one correct method for valuing an asset class. There are two methods. First, the price a normal seller would receive from a normal buyer who considers the value of the asset at maturity. And second, the price a distressed seller would accept now from a reluctant risk-averse buyer. That change in valuation methodology, unforeseen in the Bank's definition of inflation, created this crisis.

This has been confirmed by the IMF:

> "By focusing on consumer prices, Central Banks ignored risks associated with high asset prices and increased leverage."
>
> International Monetary Fund, 12 March 2009

By focusing on the wrong kind of inflation, as the Act directed it to do, the Bank of England was blindfolded to the disaster that could occur in a low inflation environment.

Never again.

.

4. CONSEQUENCES

The Myth of Inflation Targeting was the direct cause of excessive self-confidence which was itself the cause of our present predicament. This chapter describes precisely how that worked through the international banking system.

In three stunning creative leaps, huge liabilities were contained off balance sheet in the accounts of our giant global banks. This is why, when the crisis arose, it was all such a shock, and is still unravelling. This is why the banks had so many bad debts that they came to the point of collapse.

Leap No. 1: The War of the Acronym

While everyone was looking the other way, reassured by the Myth of Inflation Targeting, this economic war was started by an acronym. Here is a sentence:

> "I use CFDs in my SIV to buy CDIs in the CDS."

After a long search, it has proved impossible to find any Chairman, Chief Executive, Non-Executive Director, Shareholder or Owner of any Bank, or any Regulator or Central Banker, who can translate that sentence into plain English.

They know these acronyms exist. They know what the letters stand for. But they cannot explain what they mean.

The distinguished former Governor of the Bank of England, Eddie George, confirmed the point. Writing about these banking acronyms, he says that while he was Governor he did not understand:

> "how they were rated or related."

These are the acronyms that brought the world to its knees.

For centuries, it was thought that banks should not lend more than they had on deposit. Then it was decided that was unreasonable – too restrictive on banks. Bankers ought to be able to lend a multiple of what they had on deposit, because not all their depositors would ever ask for their money at once.

So it was agreed that banks could lend a multiple of their deposits, up to a reasonable ratio. After 1974, that ratio would be determined by a body of good men and true known as The Basel Committee on Global Financial Stability.

This Basel Committee was to be a high-powered Committee of central banks, regulators and bankers. Unfortunately, it would have no authority and no power. It would not lay down "rules". It would not pass "laws". It describes itself today as a "forum". It makes what it calls "recommendations". It duly made recommendations about what the capital adequacy ratios of banks should be – in other words, what their multiple of deposits should be. It did that in its Basel II Accords in 2004.

These accords represented a roadblock for banks, and stimulated the first example of bankers' remarkable creativity. They would get around the restrictions of the Basel Committee by creating what are called Structured Investment Vehicles (SIVs). For some reason, never adequately explained by any bank auditor, these could be contained "off balance sheet". In one leap, bank "loans" became "investments", and were no longer counted as part of a bank's "Loan Book". This transformed a Bank's capital ratio and enabled the bank to do exactly what was intended – to lend more.

Leap No. 2: "We are all Triple A now"

As banks began to lend more, they began to lend to people lower and lower down the asset scale. There is a popular misconception that these loans were to the so-called "sub-prime" market. There is no such place. Bankers are not so dim. There is "prime" (at the top), and the rest – a continuum of assets from top to bottom of the market.

In time, these extra levels of lending brought the banks to a second roadblock – the credit rating agencies.

Apparently, it is not possible for many large institutions to buy or invest in anything other than what are called "Triple-A" rated securities. Sometimes they do not want to; sometimes they are not allowed to. Either way, as bankers lent more, they reached the point where they had to overcome a second problem – the credit rating agencies were growing anxious about giving Triple A ratings to some of the banks so-called "investments".

The bankers soon found a creative way around that obstacle. They created a new industry. They approached insurance companies which, until then, had typically insured cars, houses and lives and so on, and made them an offer they could not refuse – a new profit centre, in which they could sell insurance policies for "securities" i.e. debts. The insurance companies willingly obliged. The banks were then able to go back to the rating agencies and say, “There you are. These debts now deserve Triple A ratings. Why? Because they are insured”.

The rest is history.

Leap No. 3: Computer shall speak unto computer

In 2000, while working at JPMorgan Chase, David Li published a paper in *The Journal of Fixed Income* titled “On Default Correlation: A Copula Function Approach.” (This is reprinted in Appendix 1).

The Gaussian Copula Function was a statistical method of correlating the behaviour of two or more variables, such as the likelihood of default on a debt. It provided a simple correlation number to rank multi-variable risk. This made it possible for computers to package different tranches of bonds with different credit ratings to suit different investors.

The computer model would give investors the product they wanted, custom-built. If you preferred low-yield, low-risk, long-maturity, you could have that. If you liked high-yield, higher-risk, short-maturity, it was all available – tailor-made, to your personal preference. For some time, regulators thought this was a benefit – it “spread the risk”.

The Collateralized Debt Instrument (CDI) market grew from $275 billion in 2000 to $4.7 trillion by 2006. In 2001, there was $920 billion in Credit Default Swaps (CDS) outstanding. By the end of 2007, that number was more than $62 trillion.

Of course, the Copula-powered computer model was sensitive to the risk of falling house prices – but nobody was interested. Everyone was convinced by the Myth of Inflation Targeting – certain that low inflation meant long-term growth and financial stability.

People ask why “traditional bank managers” did not apply “basic common sense”. The answer is that human beings were not involved. The Copula Function model enabled computers to execute billions of dollars of trades with other computers. The counterparty was another computer. A bank’s computers both sold the packages to other banks, and bought them from other banks. Computers were on both sides of the transaction.

The world was heading for the first truly technological crisis. But nobody knew it – not the Treasury, not the FSA, not the Bank of England, not the US Federal Reserve, not the US Treasury Secretary, not the Chancellor of the Exchequer, not even the Prime Minister or the US President. They were all victims of the Myth of Inflation Targeting, certain that low inflation meant growth and stability.

5. CONCLUSION

The UK Government's response to this pamphlet will be to echo the words of Ben Bernanke, Chairman of the US Federal Reserve, speaking at the London School of Economics in February 2009. He said there is a need for:

> "increased surveillance and more oversight of capital regulations and accounting rules."

He said these are needed on a global basis:

> "to detect and manage risk."

The British Government has said that the International Accounting Standards Board has recently been charged by the Financial Stability Forum and the G20 leaders to review its relevant standards and as a result is proposing new consolidation rules. It has said that, in addition, the Committee of Central European Bankers (CEBS) and the Financial Services Authority (FSA) are seeking greater disclosures on the impact of the financial crisis and the British Bankers Association (BBA) has supported this by offering their own recommendations. It concluded that this is an area therefore that needs to be taken on an international basis rather than the UK alone and by reassuring us that good work is already underway.

We are all touched by romantic faith in the wisdom of bodies such as the IASB, the FSF, the BBA, the CEBS, the FSA, etc.

However, the OECD said:

> "Basel II... fostered the creation of off-balance sheet vehicles."

These liabilities remain. Citigroup still has $1.2 trillion in off-balance sheet special purposes entities. To underline the point, consider what was said by a senior Government source in March 2009 about the liabilities of the banks:

> "In short, we do not know what they are worth. All the assets on the balance sheet have got to be valued to the best of our ability. Auditors have been brought in to work this out."

If that statement applied to the grocery shop in your local high street it would be sad, but understandable. When it applies to the biggest banks in the world, it is astonishing. No wonder the former head of the US Federal Reserve, Alan Greenspan, looked on as all these events unfolded with what he called:

> "shocked disbelief."

A request that banks routinely make to their customers:

> "Please show me your balance sheet"

was one with which the banks themselves could not promptly comply.

The best form of regulation of the banking sector is full disclosure. It should be a statutory requirement for banks' auditors to provide explanation and commentary on contingent liabilities contained in banks' invisible off-balance sheet investments. The Treasury should make provision by regulations requiring a bank's auditor to include a description in the notes to the bank's annual accounts of any significant sums which could become a liability for the bank, and of the circumstances in which they could become a liability.

The regulations should specify that the description should include sums invested in Structured Investment Vehicles and sums for which the bank has achieved insurance.

All these measures are worthy and will help, but they are not enough. This pamphlet reveals that the true culprit is not a regulation or an accounting standard; not a person or an institution or an industry, but an idea – the Myth of Inflation Targeting. It blinded us to how an economic catastrophe could occur during a period of low inflation. It lulled us into a false sense of security. Its confusion between the necessity and sufficiency of inflation targets led to a misunderstanding of epic proportions. When the myth was exposed, it took us all down with it.

As readers now know, none of us can rely on "Inflation Targeting" for our safety and security. It has been the largest public policy failure of our generation. It is not the guarantor of growth and stability, and never was. Why? Because it is a Myth.

Never again

.

APPENDIX A

The following article by David Li was first published in the US Journal of Fixed Income in April 2000.

This article is reprinted with permission from Institutional Investor, Inc. To view the original article, please go to www.iijfi.com

On Default Correlation: A Copula Function Approach

David X. Li

April 2000

Abstract

This paper studies the problem of default correlation. We first introduce a random variable called "time-until-default" to denote the survival time of each defaultable entity or financial instrument, and define the default correlation between two credit risks as the correlation coefficient between their survival times. Then we argue why a copula function approach should be used to specify the joint distribution of survival times after marginal distributions of survival times are derived from market information, such as risky bond prices or asset swap spreads. The definition and some basic properties of copula functions are given. We show that the current CreditMetrics approach to default correlation through asset correlation is equivalent to using a normal copula function. Finally, we give some numerical examples to illustrate the use of copula functions in the valuation of some credit derivatives, such as credit default swaps and first-to-default contracts.

1 Introduction

The rapidly growing credit derivative market has created a new set of financial instruments which can be used to manage the most important dimension of financial risk - credit risk. In addition to the standard credit derivative products, such as credit default swaps and total return swaps based upon a single underlying credit risk, many new products are now associated with a portfolio of credit risks. A typical example is the product with payment contingent upon the time and identity of the first or second-to-default in a given credit risk portfolio. Variations include instruments with payment contingent upon the cumulative loss before a given time in the future. The equity tranche of a collateralized bond obligation (CBO) or a collateralized loan obligation (CLO) is yet another variation, where the holder of the equity tranche incurs the first loss. Deductible and stop-loss in insurance products could also be incorporated into the basket credit derivatives structure. As more financial firms try to manage their credit risk at the portfolio level and the CBO/CLO market continues to expand, the demand for basket credit derivative products will most likely continue to grow.

Central to the valuation of the credit derivatives written on a credit portfolio is the problem of default correlation. The problem of default correlation even arises in the valuation of a simple credit default swap with one underlying reference asset if we do not assume the independence of default between the reference asset and the default swap seller. Surprising though it may seem, the default correlation has not been well defined and understood in finance. Existing literature tends to define default correlation based on discrete events which dichotomize according to survival or nonsurvival at a critical period such as one year. For example, if we denote

$$q_A = \Pr[E_A], \quad q_B = \Pr[E_B], \quad q_{AB} = \Pr[E_A E_B]$$

where E_A, E_B are defined as the default events of two securities A and B over 1 year. Then the default correlation ρ between two default events E_A and E_B, based on the standard definition of correlation of two random variables, are defined as follows

$$\rho = \frac{q_{AB} - q_A \cdot q_B}{\sqrt{q_A(1-q_A)q_B(1-q_B)}}. \tag{1}$$

This discrete event approach has been taken by Lucas [1995]. Hereafter we simply call this definition of default correlation the **discrete default correlation**.

However the choice of a specific period like one year is more or less arbitrary. It may correspond with many empirical studies of default rate over one year period. But the dependence of default correlation on a specific time interval has its disadvantages. First, default is a time dependent event, and so is default correlation. Let us take the survival time of a human being as an example. The probability of dying within one year for a person aged 50 years today is about 0.6%, but the probability of dying for the same person within 50 years is almost a sure event. Similarly default correlation is a time dependent quantity. Let us now take the survival times of a couple, both aged 50 years today. The correlation between the two discrete events that each dies within one year is very small. But the correlation between the two discrete events that each dies within 100 years is 1. Second, concentration on a single period of one year wastes important information. There are empirical studies which show that the default tendency of corporate bonds is linked to their age since issue. Also there are strong links between the economic cycle and defaults. Arbitrarily focusing on a one year period neglects this important information. Third, in the majority of credit derivative valuations, what we need is not the default correlation of two entities over the next year. We may need to have a joint distribution of survival times for the next 10 years. Fourth, the calculation of default rates as simple proportions is possible only when no samples are censored during the one year period[1].

This paper introduces a few techniques used in survival analysis. These techniques have been widely applied to other areas, such as life contingencies in actuarial science and industry life testing in reliability studies, which are similar to the credit problems we encounter here. We first introduce a random variable called

[1]A company who is observed, default free, by Moody's for 5-years and then withdrawn from the Moody's study must have a survival time exceeding 5 years. Another company may enter into Moody's study in the middle of a year, which implies that Moody's observes the company for only half of the one year observation period. In the survival analysis of statistics, such incomplete observation of default time is called *censoring*. According to Moody's studies, such incomplete observation does occur in Moody's credit default samples.

"time-until-default" to denote the survival time of each defaultable entity or financial instrument. Then, we define the default correlation of two entities as the correlation between their survival times. In credit derivative valuation we need first to construct a credit curve for each credit risk. A credit curve gives all marginal conditional default probabilities over a number of years. This curve is usually derived from the risky bond spread curve or asset swap spreads observed currently from the market. Spread curves and asset swap spreads contain information on default probabilities, recovery rate and liquidity factors etc. Assuming an exogenous recovery rate and a default treatment, we can extract a credit curve from the spread curve or asset swap spread curve. For two credit risks, we would obtain two credit curves from market observable information. Then, we need to specify a joint distribution for the survival times such that the marginal distributions are the credit curves. Obviously, this problem has no unique solution. Copula functions used in multivariate statistics provide a convenient way to specify the joint distribution of survival times with given marginal distributions. The concept of copula functions, their basic properties, and some commonly used copula functions are introduced. Finally, we give a few numerical examples of credit derivative valuation to demonstrate the use of copula functions and the impact of default correlation.

2 Characterization of Default by Time-Until-Default

In the study of default, interest centers on a group of individual companies for each of which there is defined a point event, often called default, (or survival) occurring after a length of time. We introduce a random variable called the **time-until-default**, or simply survival time, for a security, to denote this length of time. This random variable is the basic building block for the valuation of cash flows subject to default.

To precisely determine time-until-default, we need: an unambiguously defined time origin, a time scale for measuring the passage of time, and a clear definition of default.

We choose the current time as the time origin to allow use of current market information to build credit curves. The time scale is defined in terms of years for continuous models, or number of periods for discrete models. The meaning of default is defined by some rating agencies, such as Moody's.

2.1 Survival Function

Let us consider an existing security A. This security's time-until-default, T_A, is a continuous random variable which measures the length of time from today to the time when default occurs. For simplicity we just use T which should be understood as the time-until-default for a specific security A. Let $F(t)$ denote the distribution function of T,

$$F(t) = \Pr(T \leq t), \qquad t \geq 0 \tag{2}$$

and set

$$S(t) = 1 - F(t) = \Pr(T > t), \qquad t \geq 0. \tag{3}$$

We also assume that $F(0) = 0$, which implies $S(0) = 1$. The function $S(t)$ is called the **survival function**. It gives the probability that a security will attain age t. The distribution of T_A can be defined by specifying either the distribution function $F(t)$ or the survival function $S(t)$. We can also define a probability density function as follows

$$f(t) = F'(t) = -S'(t) = \lim_{\Delta \to 0^+} \frac{\Pr[t \leq T < t + \Delta]}{\Delta}.$$

To make probability statements about a security which has survived x years, the future life time for this security is $T - x | T > x$. We introduce two more notations

$$\begin{aligned} {}_tq_x &= \Pr[T - x \leq t | T > x], \qquad t \geq 0 \\ {}_tp_x &= 1 - {}_tq_x = \Pr[T - x > t | T > x], \qquad t \geq 0. \end{aligned} \tag{4}$$

The symbol ${}_tq_x$ can be interpreted as the conditional probability that the security A will default within the next t years conditional on its survival for x years. In the special case of $X = 0$, we have

$${}_tp_0 = S(t) \quad x \geq 0.$$

If $t = 1$, we use the actuarial convention to omit the prefix 1 in the symbols ${}_tq_x$ and ${}_tp_x$, and we have

$$\begin{aligned} p_x &= \Pr[T - x > 1 | T > x] \\ q_x &= \Pr[T - x \leq 1 | T > x]. \end{aligned}$$

The symbol q_x is usually called the *marginal default probability*, which represents the probability of default in the next year conditional on the survival until the beginning of the year. A credit curve is then simply defined as the sequence of $q_0, q_1, \cdots, q_n$ in discrete models.

2.2 Hazard Rate Function

The distribution function $F(t)$ and the survival function $S(t)$ provide two mathematically equivalent ways of specifying the distribution of the random variable time-until-default, and there are many other equivalent functions. The one used most frequently by statisticians is the hazard rate function which gives the instantaneous default probability for a security that has attained age x.

$$\begin{aligned} \Pr[x < T \leq x + \Delta x | T > x] &= \frac{F(x + \Delta x) - F(x)}{1 - F(x)} \\ &\approx \frac{f(x)\Delta x}{1 - F(x)}. \end{aligned}$$

The function

$$\frac{f(x)}{1 - F(x)}$$

has a conditional probability density interpretation: it gives the value of the conditional probability density function of T at exact age x, given survival to that time. Let's denote it as $h(x)$, which is usually called the **hazard rate function**. The relationship of the hazard rate function with the distribution function and survival function is as follows

$$h(x) = \frac{f(x)}{1 - F(x)} = -\frac{S'(x)}{S(x)}. \tag{5}$$

Then, the survival function can be expressed in terms of the hazard rate function,

$$S(t) = e^{-\int_0^t h(s)ds}.$$

Now, we can express ${}_tq_x$ and ${}_tp_x$ in terms of the hazard rate function as follows

$$\begin{aligned} {}_tp_x &= e^{-\int_0^t h(s+x)ds}, \\ {}_tq_x &= 1 - e^{-\int_0^t h(s+x)ds}. \end{aligned} \tag{6}$$

In addition,

$$F(t) = 1 - S(t) = 1 - e^{-\int_0^t h(s)ds},$$

and

$$f(t) = S(t) \cdot h(t). \tag{7}$$

which is the density function for T.

A typical assumption is that the hazard rate is a constant, h, over certain period, such as $[x, x+1]$. In this case, the density function is

$$f(t) = he^{-ht}$$

which shows that the survival time follows an exponential distribution with parameter h. Under this assumption, the survival probability over the time interval $[x, x+t]$ for $0 < t \leq 1$ is

$$_t p_x = 1 - {_t q_x} = e^{-\int_0^t h(s)ds} = e^{-ht} = (p_x)^t$$

where p_x is the probability of survival over one year period. This assumption can be used to scale down the default probability over one year to a default probability over a time interval less than one year.

Modelling a default process is equivalent to modelling a hazard function. There are a number of reasons why modelling the hazard rate function may be a good idea. First, it provides us information on the immediate default risk of each entity known to be alive at exact age t. Second, the comparisons of groups of individuals are most incisively made via the hazard rate function. Third, the hazard rate function based model can be easily adapted to more complicated situations, such as where there is censoring or there are several types of default or where we would like to consider stochastic default fluctuations. Fourth, there are a lot of similarities between the hazard rate function and the short rate. Many modeling techniques for the short rate processes can be readily borrowed to model the hazard rate.

Finally, we can define the joint survival function for two entities A and B based on their survival times T_A and T_B,

$$S_{T_A T_B}(s,t) = \Pr[T_A > s, T_B > t].$$

The joint distributional function is

$$\begin{aligned} F(s,t) &= \Pr[T_A \leq s, T_B \leq t] \\ &= 1 - S_{T_A}(s) - S_{T_B}(t) + S_{T_A T_B}(s,t). \end{aligned}$$

The aforementioned concepts and results can be found in survival analysis books, such as Bowers et al. [1997], Cox and Oakes [1984].

3 Definition of Default Correlations

The default correlation of two entities A and B can then be defined with respect to their survival times T_A and T_B as follows

$$\begin{aligned}\rho_{AB} &= \frac{Cov(T_A, T_B)}{\sqrt{Var(T_A)Var(T_B)}} \\ &= \frac{E(T_A T_B) - E(T_A)E(T_B)}{\sqrt{Var(T_A)Var(T_B)}}.\end{aligned} \tag{8}$$

Hereafter we simply call this definition of default correlation **the survival time correlation**. The survival time correlation is a much more general concept than that of the discrete default correlation based on a one period. If we have the joint distribution $f(s,t)$ of two survival times T_A, T_B, we can calculate the discrete default correlation. For example, if we define

$$\begin{aligned}E_1 &= [T_A < 1], \\ E_2 &= [T_B < 1],\end{aligned}$$

then the discrete default correlation can be calculated using equation (1) with the following calculation

$$\begin{aligned}q_{12} &= \Pr[E_1 E_2] = \int_0^1 \int_0^1 f(s,t)\,ds\,dt \\ q_1 &= \int_0^1 f_A(s)\,ds \\ q_2 &= \int_0^1 f_B(t)\,dt.\end{aligned}$$

However, knowing the discrete default correlation over one year period does not allow us to specify the survival time correlation.

4 The Construction of the Credit Curve

The distribution of survival time or time-until-default can be characterized by the distribution function, survival function or hazard rate function. It is shown in Section 2 that all default probabilities can be calculated once a characterization is given. The hazard rate function used to characterize the distribution of survival time can also be called a credit curve due to its similarity to a yield curve. But the basic question is: how do we obtain the credit curve or the distribution of survival time for a given credit?

There exist three methods to obtain the term structure of default rates:

(i) Obtaining historical default information from rating agencies;

(ii) Taking the Merton option theoretical approach;

(iii) Taking the implied approach using market prices of defaultable bonds or asset swap spreads.

Rating agencies like Moody's publish historical default rate studies regularly. In addition to the commonly cited one-year default rates, they also present multi-year default rates. From these rates we can obtain the hazard rate function. For example, Moody's (see Carty and Lieberman [1997]) publishes weighted average cumulative default rates from 1 to 20 years. For the B rating, the first 5 years cumulative default rates in percentage are 7.27, 13.87, 19.94, 25.03 and 29.45. From these rates we can obtain the marginal conditional default probabilities. The first marginal conditional default probability in year one is simply the one-year default probability, 7.27%. The other marginal conditional default probabilities can be obtained using the following formula:

$$ {}_{n+1}q_x = {}_nq_x + {}_np_x \cdot q_{x+n}, \tag{9} $$

which simply states that the probability of default over time interval $[0, n+1]$ is the sum of the probability of default over the time interval $[0, n]$, plus the probability of survival to the end of nth year and default in the following year. Using equation (9) we have the marginal conditional default probability:

$$ q_{x+n} = \frac{{}_{n+1}q_x - {}_nq_x}{1 - {}_nq_x} $$

which results in the marginal conditional default probabilities in year 2, 3, 4, 5 as 7.12%, 7.05%, 6.36% and 5.90%. If we assume a piecewise constant hazard rate function over each year, then we can obtain the hazard rate function using equation (6). The hazard rate function obtained is given in Figure (1).

Using diffusion processes to describe changes in the value of the firm, Merton [1974] demonstrated that a firm's default could be modeled with the Black and Scholes methodology. He showed that stock could be considered as a call option on the firm with strike price equal to the face value of a single payment debt. Using this framework we can obtain the default probability for the firm over one period, from which we can translate this default probability into a hazard rate function. Geske [1977] and Delianedis and Geske [1998] extended Merton's analysis to produce a term structure of default probabilities. Using the relationship between the hazard rate and the default probabilities we can obtain a credit curve.

Alternatively, we can take the implicit approach by using market observable information, such as asset swap spreads or risky corporate bond prices. This is the approach used by most credit derivative trading desks. The extracted default probabilities reflect the market-agreed perception today about the future default tendency of the underlying credit. Li [1998] presents one approach to building the credit curve from market information based on the Duffie and Singleton [1996] default treatment. In that paper the author assumes that there exists a series of bonds with maturity 1, 2, .., n years, which are issued by the same company and have the same seniority. All of those bonds have observable market prices. From the market price of these bonds we can calculate their yields to maturity. Using the yield to maturity of corresponding treasury bonds we obtain a yield spread curve over treasury (or asset swap spreads for a yield spread curve over LIBOR). The credit curve construction is based on this yield spread curve and an exogenous assumption about the recovery rate based on the seniority and the rating of the bonds, and the industry of the corporation.

The suggested approach is contrary to the use of historical default experience information provided by rating agencies such as Moody's. We intend to use market information rather than historical information for the following reasons:

- The calculation of profit and loss for a trading desk can only be based on current market information. This current market information reflects the market agreed perception about the evolution of the market in the future, on which the actual profit and loss depend. The default rate derived from current market information may be much different than historical default rates.

- Rating agencies use classification variables in the hope that homogeneous risks will be obtained after classification. This technique has been used elsewhere like in pricing automobile insurance. Unfortunately, classification techniques omit often some firm specific information. Constructing a credit curve for each credit allows us to use more firm specific information.

- Rating agencies reacts much slower than the market in anticipation of future credit quality. A typical example is the rating agencies reaction to the recent Asian crisis.

- Ratings are primarily used to calculate default frequency instead of default severity. However, much of credit derivative value depends on both default frequency and severity.

- The information available from a rating agency is usually the one year default probability for each rating group and the rating migration matrix. Neither the transition matrixes, nor the default probabilities are necessarily stable over long periods of time. In addition, many credit derivative products have maturities well beyond one year, which requires the use of long term marginal default probability.

It is shown under the Duffie and Singleton approach that a defaultable instrument can be valued as if it is a default free instrument by discounting the defaultable cash flow at a *credit risk adjusted discount factor*. The credit risk adjusted discount factor or the total discount factor is the product of risk-free discount factor and the pure credit discount factor if the underlying factors affecting default and those affecting the interest rate are independent. Under this framework and the assumption of a piecewise constant hazard rate function, we can derive a credit curve or specify the distribution of the survival time.

5 Dependent Models - Copula Functions

Let us study some problems of an n credit portfolio. Using either the historical approach or the market implicit approach, we can construct the marginal distribution of survival time for each of the credit risks in the portfolio. If we assume mutual independence among the credit risks, we can study any problem associated with the portfolio. However, the independence assumption of the credit risks is obviously not realistic; in reality, the default rate for a group of credits tends to be higher in a recession and lower when the economy is booming. This implies that each credit is subject to the same set of macroeconomic environment, and that there exists some form of positive dependence among the credits. To introduce a correlation structure into the portfolio, we must determine how to specify a joint distribution of survival times, with given marginal distributions.

Obviously, this problem has no unique solution. Generally speaking, knowing the joint distribution of random variables allows us to derive the marginal distributions and the correlation structure among the random variables, but not vice versa. There are many different techniques in statistics which allow us to specify a joint distribution function with given marginal distributions and a correlation structure. Among them, copula function is a simple and convenient approach. We give a brief introduction to the concept of copula function in the next section.

5.1 Definition and Basic Properties of Copula Function

A copula function is a function that links or marries univariate marginals to their full multivariate distribution. For m uniform random variables, U_1, U_2, $\cdots$,U_m, the joint distribution function C, defined as

$$C(u_1, u_2, \cdots, u_m, \rho) = \Pr[U_1 \le u_1, U_2 \le u_2, \cdots, U_m \le u_m]$$

can also be called a *copula function*.

Copula functions can be used to link marginal distributions with a joint distribution. For given univariate marginal distribution functions $F_1(x_1)$, $F_2(x_2)$,$\cdots$, $F_m(x_m)$, the function

$$C(F_1(x_1), F_2(x_2), \cdots, F_m(x_m)) = F(x_1, x_2, \cdots x_m),$$

which is defined using a copula function C, results in a multivariate distribution function with univariate marginal distributions as specified $F_1(x_1)$, $F_2(x_2)$,$\cdots$, $F_m(x_m)$.

This property can be easily shown as follows:

$$
\begin{aligned}
C(F_1(x_1), F_2(x_2), \cdots, F_m(x_m), \rho) &= \Pr\left[U_1 \le F_1(x_1), U_2 \le F_2(x_2), \cdots, U_m \le F_m(x_m)\right] \\
&= \Pr\left[F_1^{-1}(U_1) \le x_1, F_2^{-1}(U_2) \le x_2, \cdots, F_m^{-1}(U_m) \le x_m\right] \\
&= \Pr\left[X_1 \le x_1, X_2 \le x_2, \cdots, X_m \le x_m\right] \\
&= F(x_1, x_2, \cdots x_m).
\end{aligned}
$$

The marginal distribution of X_i is

$$
\begin{aligned}
& C(F_1(+\infty), F_2(+\infty), \cdots F_i(x_i), \cdots, F_m(+\infty), \rho) \\
= \; & \Pr\left[X_1 \le +\infty, X_2 \le +\infty, \cdots, X_i \le x_i, X_m \le +\infty\right] \\
= \; & \Pr[X_i \le x_i] \\
= \; & F_i(x_i).
\end{aligned}
$$

Sklar [1959] established the converse. He showed that any multivariate distribution function F can be written in the form of a copula function. He proved the following: If $F(x_1, x_2, \cdots x_m)$ is a joint multivariate distribution function with univariate marginal distribution functions $F_1(x_1)$, $F_2(x_2)$,$\cdots$, $F_m(x_m)$, then there exists a copula function $C(u_1, u_2, \cdots, u_m)$ such that

$$
F(x_1, x_2, \cdots x_m) = C(F_1(x_1), F_2(x_2), \cdots, F_m(x_m)).
$$

If each F_i is continuous then C is unique. Thus, copula functions provide a unifying and flexible way to study multivariate distributions.

For simplicity's sake, we discuss only the properties of bivariate copula functions $C(u, v, \rho)$ for uniform random variables U and V, defined over the area $\{(u, v)|0 < u \le 1, 0 < v \le 1\}$, where ρ is a correlation parameter. We call ρ simply a correlation parameter since it does not necessarily equal the usual correlation coefficient defined by Pearson, nor Spearman's Rho, nor Kendall's Tau. The bivariate copula function has the following properties:

(i) Since U and V are positive random variables, $C(0, v, \rho) = C(u, 0, \rho) = 0$.

(ii) Since U and V are bounded above by 1, the marginal distributions can be obtained by $C(1, v, \rho) = v$, $C(u, 1, \rho) = u$.

(iii) For independent random variables U and V, $C(u, v, \rho) = uv$.

Frechet [1951] showed there exist upper and lower bounds for a copula function

$$\max(0, u + v - 1) \leq C(u, v) \leq \min(u, v).$$

The multivariate extension of Frechet bounds is given by Dall'Aglio [1972].

5.2 Some Common Copula Functions

We present a few copula functions commonly used in biostatistics and actuarial science.

Frank Copula The Frank copula function is defined as

$$C(u, v) = \frac{1}{\alpha} \ln \left[1 + \frac{(e^{\alpha u} - 1)(e^{\alpha v} - 1)}{e^{\alpha} - 1} \right], \quad -\infty < \alpha < \infty.$$

Bivariate Normal

$$C(u, v) = \Phi_2(\Phi^{-1}(u), \Phi^{-1}(v), \rho), \quad -1 \leq \rho \leq 1, \tag{10}$$

where Φ_2 is the bivariate normal distribution function with correlation coefficient ρ, and Φ^{-1} is the inverse of a univariate normal distribution function. As we shall see later, this is the copula function used in CreditMetrics.

Bivariate Mixture Copula Function We can form new copula function using existing copula functions. If the two uniform random variables u and v are independent, we have a copula function $C(u, v) = uv$. If the two random variables are perfect correlated we have the copula function $C(u, v) = \min(u, v)$. Mixing the two copula functions by a mixing coefficient ($\rho > 0$) we obtain a new copula function as follows

$$C(u,v) = (1-\rho)uv + \rho \min(u,v), \qquad \text{if } \rho > 0.$$

If $\rho \leq 0$ we have

$$C(u,v) = (1+\rho)uv - \rho(u-1+v)\Theta(u-1+v), \quad \text{if } \rho \leq 0,$$

where

$$\begin{aligned} \Theta(x) &= 1, \quad \text{if } x \geq 0 \\ &= 0, \quad \text{if } x < 0. \end{aligned}$$

5.3 Copula Function and Correlation Measurement

To compare different copula functions, we need to have a correlation measurement independent of marginal distributions. The usual Pearson's correlation coefficient, however, depends on the marginal distributions (See Lehmann [1966]). Both Spearman's Rho and Kendall's Tau can be defined using a copula function only as follows

$$\rho_s = 12 \iint [C(u,v) - uv]dudv,$$

$$\tau = 4 \iint C(u,v)dC(u,v) - 1.$$

Comparisons between results using different copula functions should be based on either a common Spearman's Rho or a Kendall's Tau.

Further examination of copula functions can be found in a survey paper by Frees and Valdez [1988] and a recent book by Nelsen [1999].

5.4 The Calibration of Default Correlation in Copula Function

Having chosen a copula function, we need to compute the pairwise correlation of survival times. Using the CreditMetrics (Gupton et al. [1997]) asset correlation approach, we can obtain the default correlation of two discrete events over one year period. As it happens, CreditMetrics uses the normal copula function in its default correlation formula even though it does not use the concept of copula function explicitly.

First let us summarize how CreditMetrics calculates joint default probability of two credits A and B. Suppose the one year default probabilities for A and B are q_A and q_B. CreditMetrics would use the following steps

- Obtain Z_A and Z_B such that

$$\begin{aligned} q_A &= \Pr[Z < Z_A] \\ q_B &= \Pr[Z < Z_B] \end{aligned}$$

  where Z is a standard normal random variable

- If ρ is the asset correlation, the joint default probability for credit A and B is calculated as follows,

$$\Pr[Z < Z_A, Z < Z_B] = \int_{-\infty}^{Z_A}\int_{-\infty}^{Z_B} \phi_2(x, y|\rho)dxdy = \Phi_2(Z_A, Z_B, \rho) \tag{11}$$

  where $\phi_2(x, y|\rho)$ is the standard bivariate normal density function with a correlation coefficient ρ, and Φ_2 is the bivariate accumulative normal distribution function.

If we use a bivariate normal copula function with a correlation parameter γ, and denote the survival times for A and B as T_A and T_B, the joint default probability can be calculated as follows

$$\Pr[T_A < 1, T_B < 1] = \Phi_2(\Phi^{-1}(F_A(1)), \Phi^{-1}(F_B(1), \gamma) \tag{12}$$

where F_A and F_B are the distribution functions for the survival times T_A and T_B. If we notice that

$$q_i = \Pr[T_i < 1] = F_j(1) \text{ and } Z_i = \Phi^{-1}(q_i) \text{ for } i = A, B,$$

then we see that equation (12) and equation (11) give the same joint default probability over one year period if $\rho = \gamma$.

We can conclude that CreditMetrics uses a bivariate normal copula function with the asset correlation as the correlation parameter in the copula function. Thus, to generate survival times of two credit risks, we use a bivariate normal copula function with correlation parameter equal to the CreditMetrics asset correlation. We note that this correlation parameter is not the correlation coefficient between the two survival times. The correlation coefficient between the survival times is much smaller than the asset correlation. Conveniently, the marginal distribution of any subset of an n dimensional normal distribution is still a normal distribution. Using asset correlations, we can construct high dimensional normal copula functions to model the credit portfolio of any size.

6 Numerical Illustrations

This section gives some numerical examples to illustrate many of the points discussed above. Assume that we have two credit risks, A and B, which have flat spread curves of 300 bps and 500 bps over LIBOR. These spreads are usually given in the market as asset swap spreads. Using these spreads and a constant recovery assumption of 50% we build two credit curves for the two credit risks. For details, see Li [1998]. The two credit curves are given in Figures (2) and (3). These two curves will be used in the following numerical illustrations.

6.1 Illustration 1. Default Correlation v.s. Length of Time Period

In this example, we study the relationship between the discrete default correlation (1) and the survival time correlation (8). The survival time correlation is a much more general concept than the discrete default correlation defined for two discrete default events at an arbitrary period of time, such as one year. Knowing the former allows us to calculate the latter over any time interval in the future, but not vice versa.

Using two credit curves we can calculate all marginal default probabilities up to anytime t in the future, i.e.

$$ {}_tq_0 \quad = \quad \Pr[\tau < t] = 1 - e^{-\int_0^t h(s)ds}, $$

where $h(s)$ is the instantaneous default probability given by a credit curve. If we have the marginal default probabilities ${}_tq_0^A$ and ${}_tq_0^B$ for both A and B, we can also obtain the joint probability of default over the time interval $[0, t]$ by a copula function $C(u, v)$,

$$ \Pr[T_A < t, T_B < t] = C({}_tq_0^A, {}_tq_0^B). $$

Of course we need to specify a correlation parameter ρ in the copula function. We emphasize that knowing ρ would allow us to calculate the survival time correlation between T_A and T_B.

We can now obtain the discrete default correlation coefficient ρ_t between the two discrete events that A and B default over the time interval $[0, t]$ based on the formula (1). Intuitively, the discrete default correlation ρ_t should be an increasing function of t since the two underlying credits should have a higher tendency of joint default over longer periods. Using the bivariate normal copula function (10) and $\rho = 0.1$ as an example we obtain Figure (4).

From this graph we see explicitly that the discrete default correlation over time interval $[0, t]$ is a function of t. For example, this default correlation coefficient goes from 0.021 to 0.038 when t goes from six months to twelve months. The increase slows down as t becomes large.

6.2 Illustration 2. Default Correlation and Credit Swap Valuation

The second example shows the impact of default correlation on credit swap pricing. Suppose that credit A is the credit swap seller and credit B is the underlying reference asset. If we buy a default swap of 3 years with a reference asset of credit B from a risk-free counterparty we should pay 500 bps since holding the underlying asset and having a long position on the credit swap would create a riskless portfolio. But if we buy the default swap from a risky counterparty how much we should pay depends on the credit quality of the counterparty and the default correlation between the underlying reference asset and the counterparty.

Knowing only the discrete default correlation over one year we cannot value any credit swaps with a maturity longer than one year. Figure (5) shows the impact of asset correlation (or implicitly default correlation) on the credit swap premium. From the graph we see that the annualized premium decreases as the asset correlation between the counterparty and the underlying reference asset increases. Even at zero default correlation the credit swap has a value less than 500 bps since the counterparty is risky.

6.3 Illustration 3. Default Correlation and First-to-Default Valuation

The third example shows how to value a first-to-default contract. We assume we have a portfolio of n credits. Let us assume that for each credit i in the portfolio we have constructed a credit curve or a hazard rate function for its survival time T_i. The distribution function of T_i is $F_i(t)$. Using a copula function C we also obtain the joint distribution of the survival times as follows

$$F(t_1, t_2, \cdots, t_n) = C(F_1(t_1), F_2(t_2), \cdots, F_n(t_n)).$$

If we use normal copula function we have

$$F(t_1, t_2, \cdots, t_n) = \Phi_n(\Phi^{-1}(F_1(t_1)), \Phi^{-1}(F_2(t_2)), \cdots, \Phi^{-1}(F_n(t_n)))$$

where Φ_n is the n dimensional normal cumulative distribution function with correlation coefficient matrix Σ.

To simulate correlated survival times we introduce another series of random variables $Y_1, Y_2, \cdots Y_n$, such that

$$Y_1 = \Phi^{-1}(F_1(T_1)), Y_2 = \Phi^{-1}(F_2(T_2)), \cdots, Y_n = \Phi^{-1}(F_n(T_n)). \tag{13}$$

Then there is a one-to-one mapping between Y and T. Simulating $\{T_i|i = 1, 2, ..., n\}$ is equivalent to simulating $\{Y_i|i = 1, 2, ..., n\}$. As shown in the previous section the correlation between the Y's is the asset correlation of the underlying credits. Using CreditManager from RiskMetrics Group we can obtain the asset correlation matrix Σ. We have the following simulation scheme

- Simulate $Y_1, Y_2, \cdots Y_n$ from an n-dimension normal distribution with correlation coefficient matrix Σ.
- Obtain $T_1, T_2, \cdots T_n$ using $T_i = F_i^{-1}(N(Y_i)), i = 1, 2, \cdots, n$.

With each simulation run we generate the survival times for all the credits in the portfolio. With this information we can value any credit derivative structure written on the portfolio. We use a simple structure for illustration. The contract is a two-year transaction which pays one dollar if the first default occurs during the first two years.

We assume each credit has a constant hazard rate of $h = 0.1$ for $0 < t < +\infty$. From equation (7) we know the density function for the survival time T is he^{-ht}. This shows that the survival time is exponentially distributed with mean $1/h$. We also assume that every pair of credits in the portfolio has a constant asset correlation σ^2.

Suppose we have a constant interest rate $r = 0.1$. If all the credits in the portfolio are independent, the hazard rate of the minimum survival time $T = \min(T_1, T_2, \cdots, T_n)$ is easily shown to be

$$h_T = h_1 + h_2 + \cdots + h_n = nh.$$

If $T < 2$, the present value of the contract is $1 \cdot e^{-r \cdot T}$. The survival time for the first-to-default has a density function $f(t) = h_T \cdot e^{-h_T t}$, so the value of the contract is given by

[2] To have a positive definite correlation matrix, the constant correlation coefficient has to satisfy the condition $\sigma > -\frac{1}{n-1}$.

$$
\begin{aligned}
V &= \int_0^2 1 \cdot e^{-rt} f(t) dt \\
&= \int_0^2 1 \cdot e^{-r\,t} h_T \cdot e^{-h_T\,t} dt \\
&= \frac{h_T}{r + h_T} \left(1 - e^{-2.0 \cdot (r + h_T)}\right).
\end{aligned} \tag{14}
$$

In the general case we use the Monte Carlo simulation approach and the normal copula function to obtain the distribution of T. For each simulation run we have one scenario of default times $t_1, t_2, \cdots t_n$, from which we have the first-to-default time simply as $t = \min(t_1, t_2, \cdots t_n)$.

Let us examine the impact of the asset correlation on the value of the first-to-default contract of 5-assets. If $\sigma = 0$, the expected payoff function, based on equation (14), should give a value of 0.5823. Our simulation of 50,000 runs gives a value of 0.5830. If all 5 assets are perfectly correlated, then the first-to-default of 5-assets should be the same as the first-to-default of 1-asset since any one default induces all others to default. In this case the contract should worth 0.1648. Our simulation of 50,000 runs produces a result of 0.1638. Figure (6) shows the relationship between the value of the contract and the constant asset correlation coefficient. We see that the value of the contract decreases as the correlation increases. We also examine the impact of correlation on the value of the first-to-default of 20 assets in Figure (6). As expected, the first-to-default of 5 assets has the same value of the first-to-default of 20 assets when the asset correlation approaches to 1.

7 Conclusion

This paper introduces a few standard technique used in survival analysis to study the problem of default correlation. We first introduce a random variable called “the time-until-default” to characterize the default. Then the default correlation between two credit risks is defined as the correlation coefficient between their survival times. In practice we usually use market spread information to derive the distribution of survival times. When it comes to credit portfolio studies we need to specify a joint distribution with given marginal distributions. The problem cannot be solved uniquely. The copula function approach provides one way of specifying a joint distribution with known marginals. The concept of copula functions, their basic properties and some commonly used copula functions are introduced. The calibration of the correlation parameter used in copula functions against some popular credit models is also studied. We have shown that CreditMetrics essentially uses the normal copula function in its default correlation formula even though CreditMetrics does not use the concept of copula functions explicitly. Finally we show some numerical examples to illustrate the use of copula functions in the valuation of credit derivatives, such as credit default swaps and first-to-default contracts.

References

[1] Bowers, N. L., JR., Gerber, H. U., Hickman, J. C., Jones, D. A., and Nesbitt, C. J., *Actuarial Mathematics*, 2nd Edition, Schaumberg, Illinois, Society of Actuaries, (1997).

[2] Carty, L. and Lieberman, D. *Historical Default Rates of Corporate Bond Issuers, 1920-1996*, Moodys Investors Service, January (1997).

[3] Cox, D. R. and Oakes, D. *Analysis of Survival Data*, Chapman and Hall, (1984).

[4] Dall'Aglio, G., Frechet Classes and Compatibility of Distribution Functions, *Symp. Math.*, 9, (1972), pp. 131-150.

[5] Delianedis, G. and R. Geske, Credit Risk and Risk Neutral Default Probabilities: Information about Rating Migrations and Defaults, Working paper, The Anderson School at UCLA, (1998).

[6] Duffie, D. and Singleton, K. Modeling Term Structure of Defaultable Bonds, Working paper, Graduate School of Business, Stanford University, (1997).

[7] Frechet, M. Sur les Tableaux de Correlation dont les Marges sont Donnees, *Ann. Univ. Lyon, Sect. A* 9, (1951), pp. 53-77.

[8] Frees, E. W. and Valdez, E., 1998, Understanding Relationships Using Copulas, *North American Actuarial Journal*, (1998), Vol. 2, No. 1, pp. 1-25.

[9] Gupton, G. M., Finger, C. C., and Bhatia, M. *CreditMetrics – Technical Document*, New York: Morgan Guaranty Trust Co., (1997).

[10] Lehmann, E. L. Some Concepts of Dependence, *Annals of Mathematical Statistics*, 37, (1966), pp. 1137-1153.

[11] Li, D. X., 1998, Constructing a credit curve, *Credit Risk, A RISK Special report*, (November 1998), pp. 40-44.

[12] Litterman, R. and Iben, T. Corporate Bond Valuation and the Term Structure of Credit Spreads, *Financial Analyst Journal*, (1991), pp. 52-64.

[13] Lucas, D. Default Correlation and Credit Analysis, *Journal of Fixed Income*, Vol. 11, (March 1995), pp. 76-87.

[14] Merton, R. C. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance*, 29, pp. 449-470.

[15] Nelsen, R. *An Introduction to Copulas*, Springer-Verlag New York, Inc., 1999.

[16] Sklar, A., Random Variables, Joint Distribution Functions and Copulas, *Kybernetika* 9, (1973), pp. 449-460.

Figure 1: Hazard Rate Function of B Grade Based on Moody's Study (1997)

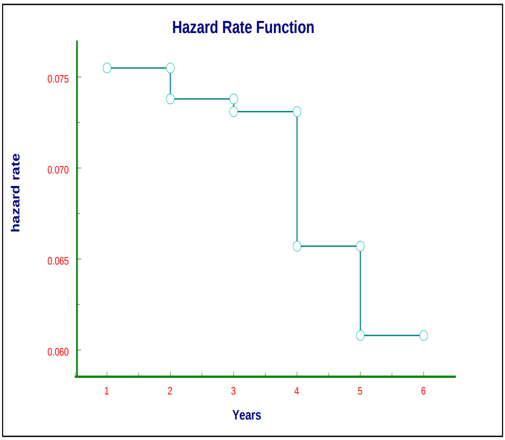


Figure 2: Credit Curve A

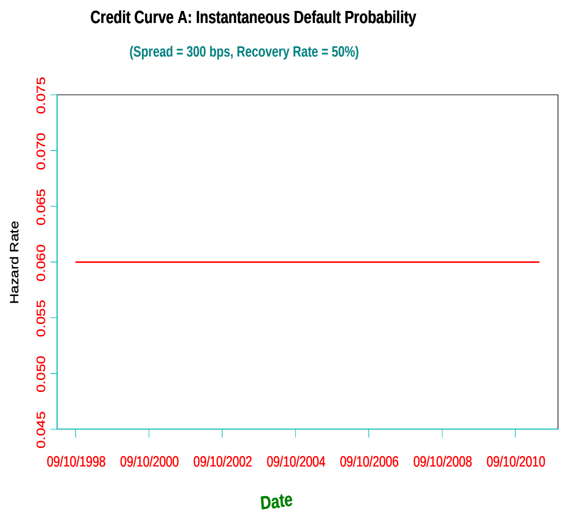


Figure 3: Credit Curve B

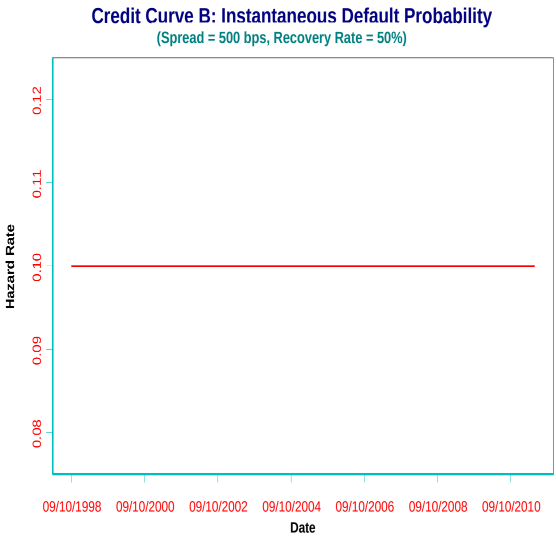


Figure 4: The Discrete Default Correlation v.s. the Length of Time Interval

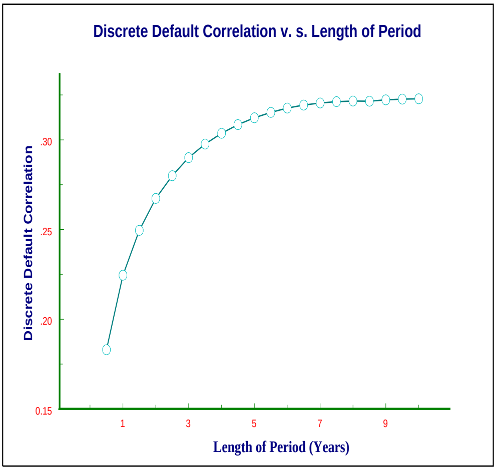


Figure 5: Impact of Asset Correlation on the Value of Credit Swap

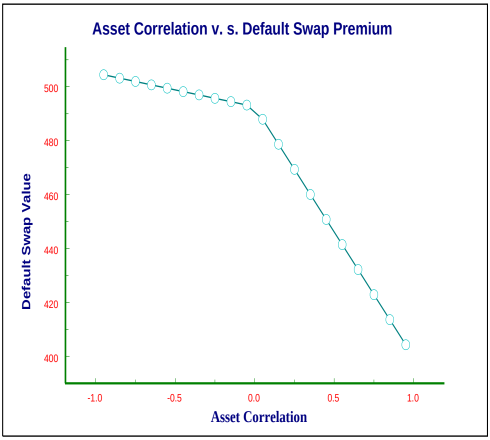
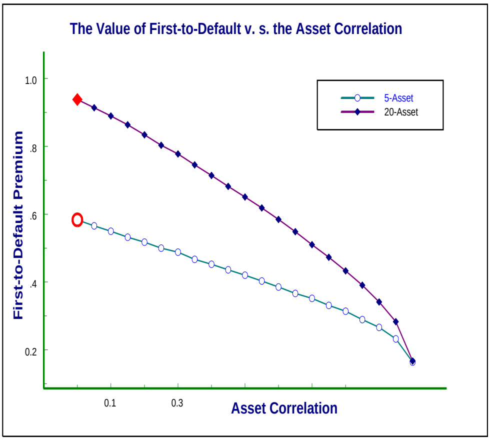


Figure 6: The Value of First-to-Default v. s. Asset Correlation



Blame for the economic crisis should not be put on greedy bankers, careless regulators, lax credit rating agencies; nor on excessive borrowing by the public.

For they were all victims of a Myth: that low inflation would bring economic growth and financial stability.

Lack of clarity between the necessity and sufficieny of inflation targets was compounded by the fact that the wrong type of inflation was being measured.

The Myth led bankers to lend more, traders to risk more, regulators to relax more, homeowners to borrow more.

But when the Myth collapsed, it took us all down with it.

Price £10.00

Centre for Policy Studies